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The Elisha Mitchell Scientific Society

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In addition to original papers on scientific subjects, this Journal publishes the Proceedings of the Elisha Mitchell Scientific Society, the Proceedings of the North Carolina Academy of Science, and the Proceedings of the North Carolina Bacteriological Society, including some abstracts of papers on scientific subjects published elsewhere.

The major portion of the work of preparing this issue was done by Dr. John N. Couch, the retiring chairman of the Editorial Board, and by Mrs. Anne Whitlock Nielsen, retiring Executive Editor.

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JOURNAL

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Elisha Mitchell Scientific Society

Volume 76 November 1960 No. 2

PROCEEDINGS OF THE FIFTY-SEVENTH ANNUAL MEETING OF THE NORTH CAROLINA ACADEMY OF SCIENCE

The fifty-seventh annual meeting of the North Carolina Academy of Science was held at Woman's College, U.N.C., Greensboro on May 6 and 7, 1960.

The Academy met in its first session at 10:30 a.m. on May 6 with President Dashiell presiding. A general session with invited papers was held on this occasion, the topic being "Basic phenomena of aging".

The business meeting of the Academy was held at 4:30 p.m. on May 6, at which time the following business was transacted:

The minutes of the 1959 meeting were approved as published in the Journal of the Elisha Mitchell Scientific Society **75:** 53–75.

Report of the Executive Committee

The Executive Committee met at the home of Dr. Wanda Hunter in Durham on April 30, 1960. At this meeting the following matters were considered:

1. All persons who had applied for membership to this date were declared elected. Names of new members appear later in the Proceedings.
2. Approved Junior Membership in the Academy for one year for ten winners in the State Science Fair and for ten winners and honerable mention in the State Science Talent Search.
3. John A. Yarbrough was named as AAAS Council representative and Academy Conference representative for the 1960 New York meetings of these two organizations.
4. Voted to allot an extra $300 to the Elisha Mitchell Journal for 1960, as was done last year. The total allotment thus will amount to 50¢ per member of the Academy plus $600 for 1960.
5. Voted to ask the editors of the Elisha Mitchell Journal for a meeting of all interested parties to consider future policies that will seek to strengthen the financial status of the Journal.
6. Voted to offer $50 to the Collegiate Academy for this year so that two Derieux Awards could be made. This was deemed necessary because the number of papers in the Collegiate Academy required division into two sections, Biological Science and Physical Science.
7. Voted to accept the invitation from North Carolina State College to meet there on May 5 and 6, 1961.
8. Voted to ask the President of the Academy to appoint a special committee to propose revision of the constitution in the light of requirements of the Federal Income Tax statutes relative to obtaining tax exemption for the Academy. The incoming President later appointed R. Harkema, V. A. Greulach and John Yarbrough on the Committee with the latter serving as chairman.
9. Voted to approve plans for a National Science Foundation sponsored science education project for 1960–61. H. E. Speece will again serve as director of the NSF project.
10. The following were elected to membership in the Academy or reinstated to active membership:

Adams, David A., N. C. State College, Raleigh

Adams, Mrs. Julia S., Taylorsville High School, Taylorsville

Allen, Eldon Perry, N. C. State Highway Comm., Raleigh

Arundel, Dr. Edna, Woman's College of U. N. C., Greensboro

Beck, Dr. Robert C., Wake Forest College, Winston-Salem

Brecht, Dr. E. A., University of N. C., Chapel Hill

Burton, Dr. Paul R., Burnsville

Byrne, Dr. William L., Duke University, Durham

Casanova, Richard L., Paleontological Research Laboratory, Statesville

Clebson, Edward E. C., U. of Tennessee, Knoxville, Tenn.
Crawford, Eugene C., Jr., Duke Medical Center, Durham
Davis, Dr. Graham J., East Carolina College, Greenville
Downey, Miss Maureen E., Durham
Duncan, Emory H., Troy High School, Troy
Edwards, Dr. Cecile H., A. and T. College, Greensboro
Elkan, Dr. Gerald Hugh, N. C. State College, Raleigh
Ely, Dr. Ralph L., Jr., Isotope Development Laboratory, Durham
Fortune, Mrs. Reva, Greensboro
Gibbons, William Edward, Catawba College, Salisbury
Grannell, Dana B., N. C. State College, Raleigh
Greene, Nathan Doyle, N. C. State College, Raleigh
Harrington, Dr. John W., Chapel Hill
Heath, Dr. Robert Winship, Woman's College of U. N. C., Greensboro
Higgins, Robert Price, Duke University, Durham
Hommersand, Dr. Max Hoyt, University of N. C., Chapel Hill
Howard, Claude C., Sanford
Hudson, Robert Clarence, Woman's College of U. N. C., Greensboro
Hunter, Dr. Hugh W., Research Triangle Institute, Durham
Kennel, Robert P., New Bern
Kimpton, George A., Mooresville Senior High School, Mooresville
Lammers, Dr. W. T., Davidson College, Davidson
Lopushinsky, William, Duke University, Durham
Lutz, Paul E., University of N. C., Chapel Hill
Maddrey, James Garland, Jr., Raleigh
Martin, David H., N. C. State College, Raleigh
McCoart, Richard F., University of N. C., Chapel Hill
McCracken, Ralph J., N. C. State College, Raleigh
McIntosh, Robert, Charles Pfizer Chemical Co., Greensboro
Miller, Don Curtis, Duke University. Durham
Miser, Dr. Hugh J., The Research Triangle Institute, Durham
Mobley, Mrs. C. Wade, Flora Macdonald College, Red Springs
Newton, Dr. Margaret, Atlantic Christian College, Wilson
Parker, Mrs. Mary Towe, Woman's College of U. N. C., Greensboro
Pati, Dr. Nityananda, A. and T. College, Greensboro
Perry, Dr. Thomas Oliver, N. C. State College, Raleigh
Rees, George H., U. S. Biological Laboratory, Beaufort
Scappucci, Mrs. Charlotte, Charlotte Country Day School, Charlotte
Schofield, Wilfred Borden, Duke University, Durham
Schulz, Richard Lee, Western Carolina College, Cullowhee
Shahdan, Peter, N. C. State College, Raleigh
Shipton, C. O., Curry School, W. C. U. N. C., Greensboro
Simmons, Mrs. Dorothy Underdown, Lenoir Rhyne College, Hickory
Smith, Lloyd B., Jr., Elon College, Elon College
Smith, Dr. T. A., Duke University, Durham
Smith, William Julius, N. C. Pharmaceutical Association, Chapel Hill
Stallings, Mrs. Rosalynd F., Spring Hope
Struble, Dr. Raimond A., N. C. State College, Raleigh
Taylor, J. D., Denoyer-Geppert Co., Charlotte
Watson, George Carson, N. C. State College, Raleigh
Whiteside, John H., The Lilly Co., High Point
Wilson, Dr. John Eric, University of N. C., Chapel Hill
Wilson, William F., Div. of Mineral Resources, Raleigh

Patron members as of July 1, 1960

Governor Luther H. Hodges, Raleigh
W. Y. Preyer, Greensboro
C. M. Goethe, Sacramento, California
Richard E. Thigpen, Charlotte
Harold D. Cooley, House of Representatives, Washington, D. C.
Justice Emery B. Denny, Raleigh
Reuben B. Robertson, Asheville

C. P. Street, Charlotte
Irving E. Carlyle, Winston-Salem
Edward R. Zane, Greensboro
Thomas Pearson, Asheville
F. R. Darkis, Durham

Sustaining members as of July 1, 1960

E. I. du Pont de Nemours & Company, Kinston
The Dickson Foundation, Inc., Mount Holly
Liggett & Myers Tobacco Company, Durham
Celanese Corporation of America, Charlotte
Acme Spinning Company, Belmont
North Carolina Pharmaceutical Foundation, Inc.
The American Tobacco Company, New York
J. P. Stevens & Co., Inc., Greensboro

The above reports and actions of the Executive Committee were approved by the Academy in its business session on May 6, 1960.

TREASURER'S REPORT

Financial Statement as of July 5, 1960

Receipts—Academy proper

Checking account adjusted balance on July 1, 1959 (Security National Bank, Raleigh)	$2472.01
Dues from memberships, regular	1686.50
Dues from patron members	170.00
Dues from sustaining members	800.00
Collegiate Academy gifts	125.00
Collegiate Academy funds deposited	50.12
Derieux Prize in Collegiate Academy	50.00
N. C. Psychological Association, for programs	8.00
Exhibit fees, commercial	20.00
AAAS, Research Grant Funds	240.00
Total Receipts, Academy proper	$5621.63

Receipts, Science Fair Program

Entry Fees from seven districts	$1450.00
Medical Society of N. C.	100.00
Medical Auxiliary, Medical Society of N. C.	50.00
American Chemical Society, N. C. Section	75.00
N. C. Society of Bacteriologists	50.00
Carolina Power and Light Company	175.00
Southern School Supply Company	157.20
J. D. Taylor for Denoyer-Geppert Company	50.00
Carolina Biological Supply Company	50.00
Cardinal Products, Inc.	100.00
N. C. Pharmaceutical Association	100.00
N. C. State Fair	600.00
Total Receipts, Science Fair Program	$2957.28
Total Receipts for Academy and Science Fair Program	$8578.91

Expenditures—Academy proper

Derieux Awards, two	100.00
Cancas and other Collegiate Academy expense	117.80
Research Grants made	1120.55
Postage, Stationery, Office supplies	145.99
Expenses for annual meeting, 1960	110.30
Printing	128.25
Secretarial Aid	167.31
Secretary's honorarium	199.10
Secretary's travel	92.53
Telephone long distance calls	50.03
Expense, representation to AAAS Convention	100.00
Academy Conference	10.92
Reprints, 1959 Proceedings	5.00
Elisha Mitchell Journal	870.50
Bad check for dues	3.00
Deposit in Savings Account, Raleigh Building and Loan	1000.00
Academy costs on 1959 NSF grant for Science Education	1000.25
Academy costs on 1960 NSF grant	105.58
Total Expenditures for Academy proper	$5327.11

Expenditures—Science Fair Program

Announcement brochure, 1960 Fairs	$157.28
Entrance Fee, National Fair	175.00
To N. C. State College, expense for State Science Fair	200.00
Prizes at State Science Fair	220.00
Expense for National Fair party, four persons, to Indianapolis	653.18
Luncheon and travel for 1961 Fairs planning session	81.71
Travel allowances, N. C. State Fair exhibitors	442.00

Remainder returned to N. C. State Fair		158.00
Miscellaneous expense		34.75
Total expenditures for Science Fairs		$2121.92
Total expenditures for Academy and Science Fairs		$7449.03
Bank balance as of July 5, 1960	$2025.38	
Less outstanding checks	895.50	
	$1129.88	
Adjusted bank balance as of July 5, 1960		$1129.88
Total to account for		$8578.91

Total Assets

Checking account, adjusted	$1129.88
Raleigh Savings and Loan Account as of July 5, 1960	1395.10
Total Assets	$2524.98

On the 6th day of July, 1960, we, the undersigned examined this account and found it to be in order.

J. M. CLARKSON, *Chairman*
F. B. MEACHAM
ARTHUR KELMAN

THE POTEAT AWARD

The award for this year was given to the best paper presented before the Mathematics Section. The award was won by Dr. Wesley O. Doggett of North Carolina State College for his paper, "An infinite set of elementary functions". A certificate will be presented at the Academy banquet next year, 1961, at the meeting in Raleigh.

REPORT OF THE COMMITTEE ON RESEARCH GRANTS

The committee again contacted departments of science in colleges and universities throughout the State, as was done last year, inviting undergraduates particularly to request research grants. The results are again discouraging. Therefore grants made in 1959–60 were largely to graduate students in the early stages of their research projects. Two deadlines for receipt of requests were set. Nine grants were made totaling $1120.55. The largest was for $200 and the smallest for $60.

M. K. BERKUT
C. E. JENNER
J. M. PARKER, III
ERNEST BALL, *Chairman*

REPORT OF THE CONSERVATION-LEGISLATIVE COMMITTEE

With the resignation of Dr. E. M. Lowry, the committee was reorganized this past year with Dr. C. F. Korstian of Duke University, Dr. T. L. Quay of N. C. State College, and Dr. R. L. Wyatt of Wake Forest College as members, and Dr. A. E. Radford of the University of North Carolina as chairman.

The committee met once during the past year. Further communication was carried on by letter and telephone. Two trips were made to prospective areas to be preserved. The committee's goals will be the same as those given in the 1958–1959 report.

The white pine area under consideration for preservation has *not* been preserved. This project is being actively pushed.

The Conservation-Legislative Committee plans to continue its efforts to have set aside natural areas within this state.

C. F. KORSTIAN
T. L. QUAY
R. L. WYATT
A. E. RADFORD, *Chairman*

REPORT OF THE COLLEGIATE ACADEMY

The seventh year of the Collegiate Academy was highlighted by the following activities:

1. Fall field trip to Duke University Marine Laboratory on November 14, 1959. This trip was arranged by Tom Hopkins, President, and Dr. Robert Haubrick, co-sponsor of the Collegiate Academy. Forty-four students and staff members from six colleges were in attendance: Atlantic Christian, Duke University, East Carolina College, Mount Olive Junior College, Pfeiffer College, and Woman's College. After an introductory session dealing with the laboratory and the habitats in the Beaufort area, various field trips were held followed by a laboratory examination of the various specimens collected.

2. Lectureship Program. Twenty-three speakers lectured at ten colleges throughout the state. This program enables any college in the state to request the services of speakers from a list

sent to the schools at the beginning of the academic year. We wish to thank the administrative officers of Duke University and the Greater University of North Carolina for providing travel funds for the speakers. This program will be continued another year.

3. One copy of CANCAS was published in February, 1960. It included details of the 1960 annual meeting, events of the last year, and the Derieux prize winning paper by Tom Hopkins, East Carolina College.

4. Annual meeting, May 6, 1960 at Woman's College. An unprecedented number of research papers necessitated scheduling two sessions, meeting simultaneously, and the awarding of two Derieux Prizes. Eight papers were presented in each section. The Derieux prizes were awarded to Miss Pat Rose, Woman's College, in the Biological Science Section, and J. S. Humphrey, Duke University, in the Physical Science Section. Other high points included a business meeting, election of officers, a luncheon, and an evening social hour.

5. Membership. The number of club and individual memberships reached the highest level since the beginning of the Collegiate Academy. There are 17 club memberships and 27 individual memberships.

In summary, this past year has been highly successful for the Collegiate Academy.

F. JOHN VERNBERG

NORTH CAROLINA SCIENCE FAIR PROGRAM

This report includes only general statements concerning the operations of the Science Fair Program in North Carolina during the period 1 May 1959 to 1 May 1960.

The same general program of past years was followed. This included the holding of seven district fairs, the promotion of local fairs, the holding of the state fair, and the sending of two winners to the National Fair. At the National Fair Miss Nancy Lawson won a first place award in the Biological Division. One notable feature of the program for the past year was the affiliation of the South Piedmont Fair at Charlotte with the National Fair. This enabled that fair to send two winners directly to the National Fair.

Again there was an increase in local fair participation although it is impossible to obtain figures on local and school fairs. Participation in the district fairs showed a considerable increase although at this time the exact figures have not been obtained from the district directors. The State Fair was held at the College Union at North Carolina State College.

I wish to express my appreciation to the directors of the District Fairs for their fine work and I wish to express my appreciation in particular to Dr. R. H. Loeppert for his fine work in connection with the State Fair. Dr. Yarbrough did his usual fine job in connection with the lining up of sponsors and in helping with the finances of the program.

I find it necessary at this time to state that I will not be able to continue my affiliation with the science fair program. Increase in other duties necessitates the elimination of certain other work. I wish to express to the Academy my thanks for the cooperation given to me and I wish to state that I have enjoyed my connection with the fair program.

H. D. CROCKFORD, *Director*

SIXTH ANNUAL NORTH CAROLINA SCIENCE TALENT SEARCH

Eight hundred eighty-three Science Talent Search examinations were requested by one hundred and four science teachers in North Carolina this past year. Of this number, one hundred and twenty-three were completed and returned to the national contest. These in turn were forwarded to the Director and the state winners selected by a committee consisting of Arthur Kelman, Ernest Beal, Henry Shannon and Herbert Speece, the Director.

North Carolina had one national winner and three honorable mentions. In the State contest four were adjudged winners and six honorable mention.

Seven scholarships were awarded to 1959–60 entrants in the North Carolina Science Talent Search by the College Foundation, Inc. Seven already in college were renewed for another year.

Two of the entrants, Charlene Markunas and Joseph P. Straley Jr., read their papers at the annual Academy of Science meeting in Greensboro.

HERBERT SPEECE, *Director*

NSF-ACADEMY INSTITUTES

The North Carolina Academy of Science, under a grant from the National Science Foundation, conducted seven in-service Institutes for Science Teachers during the 1959–60 year. They were located at Enka, Fayetteville, Nashville, Salisbury, Southern Pines, Statesville and Wallace-Rose Hill. The institutes met one night a week

for six weeks with lectures on fundamental biological and physical science. The lecturers were: Edward Horne, Terry Johnson, Harold Lewis, and Pelham Wilder from Duke; Horace Crockford, Claiborne Jones, Samuel Knight, Everett Palmatier and Joseph Straley from UNC; Thomas Turner, Phillip Hamrick, John Davis and Raymond Wyatt of Wake Forest College; Thomas Daggy and John Hopkins of Davidson College; Daniel Grosch, Arthur Kelman, Ben Smith, Wesley Doggett, Raymond Murray and Arthur Waltner of N. C. State College.

It was felt by both teacher participants and lecturers that the institutes were effective in bringing up-to-date science information to the participants, in affording an opportunity for them to get answers to questions on science, in inspiring and encouraging further science study and investigation and in promoting closer contact and cooperation between the scientists of the state and the high school science teachers.

HERBERT SPEECE, *Director*

REPORT OF THE ACADEMY CONFERENCE

The 1959 meeting of the Academy Conference was held during the AAAS meetings in Chicago on December 28, 1959. John A. Yarbrough and Charles M. Allen represented the N. C. Academy of Science. The morning session was taken up in routine business and reports from standing committees. Representatives from various Academies reported as usual on outstanding activities of their respective Academies during the year 1959. New officers elected were: Dr. Robert C. Miller of the California Academy as President-Elect for 1960; Dr. E. Ruffin Hones of the Florida Academy will continue as Secretary-Treasurer. Dr. John G. Arnold, Jr. of the New Orleans Academy will serve as President of the Academy Conference during 1960.

During the morning session there was considerable discussion of the NSF grants which have been made to various Academies. The afternoon session was given over to the formal discussion of various problems of operating the Academy programs. The previous afternoon had been used in discussing the further problems of the Junior Academy program. At the traditional dinner which is given by the AAAS to representatives from Academies, Dr. A. M. Winchester of the Florida Academy, who served as President, during 1959 made an address on the problems of authors of scientific textbooks.

REPORT ON COUNCIL MEETINGS OF THE AAAS

Two meetings of the Council of the AAAS were held in Chicago on December 27 and December 30, 1959. At the first session it was reported that Thomas Park had been chosen by mail ballot as the new President-Elect of the AAAS. Harrison Brown, Alfred Romer and Don K. Price had been chosen to the Board of Directors, the latter to fill in the unexpired term of Dr. Thomas Park.

Other than the usual business and reports of standing committees that are actually of minor interest to our Academy members, the major item discussed in the Chicago meetings dealt with the effort of the Council to decide how it might take a greater role in affairs of the Association. A special committee had been appointed to work out methods for doing this but when its report was made great difficulty was experienced in arriving at a suitable statement. After an exceedingly hot parliamentary hassle it was finally decided to continue the committee and take initial steps in accepting its report. The real issue remains to be decided.

JOHN A. YARBROUGH, *Representative*

REPORT OF THE RESOLUTIONS COMMITTEE

Be it resolved that the North Carolina Academy of Science express its appreciation to Dr. Gordon W. Blackwell, Chancellor, to the faculty and to the students of Woman's College, UNC, for their cordial hospitality at this the 57th annual meeting of the Academy. To the chairman and members of the local committee on arrangements and to the numerous persons who have cooperated with them we are grateful for the foresight and diligent efforts which have made this meeting proceed smoothly and comfortably in most pleasant surroundings. Be it further resolved that a copy of this resolution be incorporated into the minutes of the Academy for 1960 and that copies be sent to Chancellor Blackwell and to the chairman of the Committee on Arrangements at Woman's College.

ROBERT P. HOLLAR

The above resolution was unanimously adopted by the Academy.

Memorials were read to the following members: William Woodrow Martin and Monroe Craig Yoder. The memorials were approved by a rising vote of the members.

WILLIAM WOODROW MARTIN

William Woodrow Martin, Professor Emeritus of Psychology at the Woman's College, and a Life Member of the North Carolina Academy of Science, died in Greensboro on June 7, 1959 in his eighty-fifth year. After forty-seven years in the classroom as a teacher, he had retired at the age of seventy in 1944 to a quiet life in the city where he had instructed the youth for twenty-two years.

Surviving members of Mr. Martin's family include a brother, sister, and a son, William Warren Martin, all of Chicago, a daughter, Mrs. Harper J. Elam, Jr. of Greensboro, and two grandchildren. The mother of his children, Mrs. Katherine Mavity Martin, died as a result of an automobile accident in 1934. She, like Professor Martin, was in her own right a great contributor to the development of education in North Carolina and in other states where they had worked together.

Professor Martin was a native of Green Valley, Illinois, and received his advanced education at the Illinois State Normal University and at the University of Chicago, from which he received the Ph.B. and M.A. degrees. While at the University of Chicago, he studied under the great champion of a better world through education, Charles H. Judd, and it was probably from Judd that he caught the inspiration which led Professor Martin during all his professional life as a college teacher to be an advocate of better training of teachers, especially through the study of Educational Psychology.

Mr. Martin came into his field of study, Psychology, while this science was definitely still in its swaddling clothes. But he had the spirit of the scientist himself and fitted happily into the new approaches as Psychology in America grew steadily toward adult stature among its sister sciences. His acceptance of the scientific spirit was reflected in a notice he sent on an occasion to members of the Science Club at the Woman's College: "As Whitehead's aphorism cogently states, 'a clash of systems is not a disaster—it is an opportunity,' so in the spirit of science, that is of frank, open, unbiased, critical, penetrating search for truth, let us compare our several views, even though they conflict in some respects. Out of such an exchange, . . . can come only clearer and larger comprehension." True to this challenge, Mr. Martin was tireless and penetrating in his efforts to think through problems in his field. He was especially interested in making a clear and persuasive presentation of psychology from the organismic point of view. He left a legacy of works published in the Psychological Review, and in the hands of his colleagues through carefully prepared papers especially on basic concepts in Psychology. His interest in the formulation of concepts and the exposition of the implications they held for psychology and for life led him to an intense interest in and study of semantics.

Mr. Martin was a versatile man and he made significant contributions outside of the classroom. At one time he was President of the Chamber of Commerce in the town where he taught in Missouri. He is remembered on the Woman's College campus for his efforts toward establishing the Faculty Science Club Scholarship which is presented each year to a worthy student in the field of science. He was noted for his support of recreational activities such as baseball and football. He supported the effort to establish a Unitarian Fellowship in Greensboro and was one of the charter members of that local organization.

In a broader professional field away from the campus, Mr. Martin was recognized as a Life Member of the Americal Psychological Association and of the North Carolina Academy of Science. During his active professional career, he had served faithfully as a member of the American Association of University Professors and of Phi Delta Kappa. Both of these organizations honored him with Emeritus membership upon his retiring.

From these comments, it will be clear that Mr. Martin was not a professor bound to his classroom; his interests were intense in many aspects of the country and community, and he maintained these interests through all his days. He had a vital interest in public affairs, and he was not a stranger to politics. The professional status of teachers, both in the public schools and the universities and colleges, was a matter of great concern to him. He gave much attention to the relationships of management and labor, and to the significance of the capitalistic system based on the profit principle. Since he did have these broad interests and was a man of strong convictions, firm principles, and abounding courage, it is not strange to hear he was not aloof from controversy. But he was a constructive thinker who often challenged others to clearer thinking and more reasonable statements of their positions. He

was a man of great intellect, an open-hearted, free-spoken espouser of causes.

In his private life, Mr. Martin was always the thoughtful gentleman. Friendship with him was a rich, comfortable, and often colorful experience. He was a stalwart man always, whether as a teacher, a scientist, or as a friend. Many of his younger colleagues will remember him especially as the one who lent a strong hand in troublesome days.

KEY L. BARKLEY

MONROE CRAIG YODER

The death of Dr. M. Craig Yoder on May 21, 1959 brought to a close a long period of years as student and valuable teacher of biology. A native of Catawba County, North Carolina, born in 1893, Dr. Yoder was graduated from Lenoir Rhyne College at Hickory, North Carolina, in 1917 with the A.B. degree. He continued his studies at the University of North Carolina; Duke University; the Sorbonne, Paris; the Marine Fisheries Laboratory, Beaufort, North Carolina; and at the University of Virginia, where he received the Master of Arts degree in 1924. He was granted the Doctor of Science degree by Lenoir Rhyne College April 15, 1959.

After graduating from Lenoir Rhyne College, he taught briefly there until he enlisted with the U. S. armed services with which he served overseas from 1917 until 1919. In 1921 he returned to Lenoir Rhyne College to establish the Department of Biology, and he served continuously from that time as professor and head of the department. During the years of World War II, in addition to his duties as professor of biology he instructed in Meteorology in the training of naval aviation cadets for two years and then was coordinator in charge of the cadet training program at Lenoir Rhyne College until the end of the war.

Dr. Yoder was active in church and in community affairs. He was a member of St. Andrew's Lutheran Church of Hickory, North Carolina, and served this church in a number of capacities, including that of teacher of the Men's Bible Class for 32 years and several terms as Sunday School Superintendent. He also served as vice-president and member of the Board of Directors of the Lutheran Brotherhood of the North Carolina Synod.

In community affairs, Dr. Yoder was a charter member of the Catawba Valley Executive's Club, and served on the board of directors for this club. A member of the Hickory Kiwanis Club he served as director and as president of the local club and for one term was Carolina's District Lieutenant Governor of the Second Division. Working with the Boy Scout Organization for Hickory and the surrounding area he served for many years as examiner in nature studies and other sciences.

To his students and associates, Dr. Yoder was an example of Christian character and devotion; and because of his instruction, guidance, and inspiration many men and women have gone forth into wider fields of service associated with the sciences.

A member of Phi Beta Kappa, Dr. Yoder was also a member of various other professional organizations including the North Carolina Academy of Science, the Association of Southeastern Biologists, the American Association for the Advancement of Science, the American Chemical Society, and the American Association of University Professors. He is listed in American Men of Science.

REPORT OF THE NOMINATING COMMITTEE

The committee, consisting of G. R. MacCarthy, J. N. Couch and P. M. Ginnings submitted the following nominations:

President: H. W. Jensen, Warren-Wilson College

Vice-President: E. G. Purdom, Guilford College

Member of Executive Committee: Mrs. Pauline M. Longest, Fayetteville

Member of Research Grants Committee: V. M. Cutter, Jr., Woman's College, U.N.C.

The above nominees were unanimously elected by the Academy. This concluded the business and the meeting was adjourned.

More than 110 members of the Academy, guests and friends enjoyed the Annual Dinner of the Academy in the Ballroom of Elliott Hall at 6:15 p.m. Vice-President Wanda Hunter presided at the dinner and the brief session which followed. The 1959 Poteat Award, presented annually by Phipps & Bird, Inc. of Richmond, Virginia was formally presented to G. R. MacCarthy for his paper in 1959 in the Geology section on "The energy released during formation of joints." Because of the large number of papers presented in the Collegiate Academy session two awards were made this year, one in Biological Science and one in Physical Science. Pat Rose of Woman's College, U.N.C. was the

winner of the award in Biological Science with her paper on "Correlation between penicillinase production and bacteriophage adsorption." J. S. Humphrey of Duke University was the winner in the Physical Science section.

Following the banquet, the members and friends adjourned to the Legislature Room in Elliott Hall where President Dashiell presented his Presidential Address on "The animal in his world of space." Vice-President Hunter presided and presented Dr. Dashiell to the audience. Following the lecture all members and guests were entertained at a very pleasant social hour through the courtesy of Woman's College.

The following officers were elected for the respective sections:

Biochemistry and Physiology: Chairman, G. Howard Satterfield; Secretary, Gladys W. Royal

Botany: Chairman, Harold Evans; Secretary, Jane Philpott

Geology: Chairman, S. G. Conrad; Secretary, S. Duncan Heron, Jr.

Mathematics: Chairman, E. F. Canaday; Secretary, Jean B. Mobley

Physics: Chairman, J. I. Hopkins; Secretary, T. J. Turner

Zoology: Chairman, Charles Allen; Secretary, John Vernberg

High School Teaching: Chairman, C. O. Shipton; Secretary, Mrs. Sue Phillips

The following constitute the personnel of the standing committees:

Executive: H. W. Jensen, E. G. Purdom, John A. Yarbrough, Paul J. Kramer, Jesse P. Tyndall, and Mrs. Pauline Longest

Research Grants: M. K. Berkut, C. E. Jenner, John M. Parker, III and V. M. Cutter, Jr.

At this writing Dr. Jensen, President of the Academy has appointed the Auditing Committee consisting of J. M. Clarkson, Frank Meacham, and Arthur Kelman. Other committees will be appointed in the early fall of 1960 for the school year of 1960–61.

The following papers were presented during the meetings at Greensboro. Those marked with an x are abstracted in these proceedings:

GENERAL SESSION

Basic Phenomena of Aging

Aging in Rotifera. Albert I. Lansing, Department of Anatomy, School of Medicine, University of Pittsburgh

Extrinsic factors in "age" changes. Norman M. Sulkin, Department of Anatomy, Bowman Gray School of Medicine

Aging, cerebral metabolism, and mental function. Walter D. Obrist, Department of Psychiatry, Duke University School of Medicine

COLLEGIATE ACADEMY

Section A. Biological Science

The relation between temperature and flashing intervals in adult male fire-flies, Photinus pyralis (L.). Leland N. Edmunds, Davidson College

Possible relations of environment to suranal plate and genital pore variation in Arbacia punctulata (Lam.). Thomas Hopkins, East Carolina College

Studies of diatom distributions in the Pawley's Island, South Carolina region. Nell Parsons, Queens College

Correlation between penicillinase production and bacteriophage adsorption. Pat Rose, Woman's College, U.N.C.

Antigenic properties of dwarf colonies of Salmonella typhi. Juliana Trimble, Guilford College

Comparative growth rates of wild types and carotenoid mutants in Sporobolomyces salmonicolor. Joanne Yundt, Woman's College, U.N.C.

An ultrasonic generator; construction and application to insect material. George W. Brown, N. C. State College

Silicic acid chromatography of hypothalmic lipids. Paul L. Keyes, N. C. State College

Section B. Physical Science

Phosphorus in Linsley Pond sediments. John C. Boykin, Duke University

A negative current integral counter for a Van de Graaf electrostatic accelerator. Larry W. Brown, U.N.C.

Strain-amplitude-dependent internal friction of pure silver. Larry Fabian, Wake Forest College

The occurrence and significance of the Miocene guide fossil Ecphora *in the Atlantic Coastal Plain.* Benjamin A. Morgan, III, U.N.C.

Perturbation integrals for flux-trap reactors. John W. McCrary, N. C. State College

Diffusion of water into and from wood under controlled temperature and humidity. Ralph M. Nelson, N. C. State College

A photographic study of mixing in diffusion cells. Samuel C. Winchester, Jr., N. C. State College

BIOCHEMISTRY AND PHYSIOLOGY SECTION

x *Early incorporation of l-methionine-methyl-C^{14} and dl-methionine-2-C^{14} and radioactive compounds in tissue fractions of adult rats.* Cecile H. Edwards, Evelyn L. Gadsden, Gerald A. Edwards, and Evans Booker, A. and T. College

x *Appearance of radiocarbon from l-methionine-methyl-C^{14} and dl-methionine-2-C^{14} in heme, globin, urine, expired carbon dioxide, tissues and tissue proteins of rats at selected time intervals.* Evelyn L. Gadsden, Cecile H. Edwards, Gerald A. Edwards and Evans Booker, A. and T. College

x *Formation of phosphatidyl dimethylethanolamine in liver preparations.* C. Artom, Bowman Gray School of Medicine

The occurrence and function of specific phosphatases. William L. Byrne, Duke University

Electrophoresis of histones. Claude L. McClure and J. L. Irvin, U.N.C.

x *Early donor-host considerations in radiation protection by tissue transplants. I. Serotonin level in bone marrow and fetal liver transplants as a factor in radiation protection.* Gladys W. Royal and George C. Royal, Jr., A. and T. College

x *Early donor-host consideration in radiation protection by tissue transplants. II. Formation of hemagglutinins in tissue homogenates from irradiated mice after heterologous bone marrow transplantation.* George C. Royal, Jr., and Gladys W. Royal, A. and T. College

x *Dietary and therapeutic studies on spontaneous atherosclerosis in pigeons.* H. B. Lofland and T. B. Clarkson, Bowman Gray School of Medicine

x *Non-dynamic state in mammalian tisues.* John E. Wilson, U.N.C.

x *Some studies on the mechanical aspects of panting.* Eugene C. Crawford, Jr., Duke University

x *Effects of anti-inflammatory drug in canine distemper.* N. Pati, A. and T. College

x *Trypsin and blood clotting.* J. H. Ferguson, S. G. Iatridis, E. G. Wilson and H. A. Rierson, U.N.C.

BOTANY SECTION

Photosynthesis and respiration measurements in Trisetum spicatum, *an arctic-alpine grass.* E.E.C. Clebsch and W. D. Billings, Duke University

The ecotone between spruce-fir and deciduous forest in the Great Smoky Mountains. Wilfred Schofield, Duke University

A botanical reconnaissance of Portsmouth Island, N. C. C. John Burk, U.N.C.

x *Two old specimens of American chestnut in North Carolina.* H. Branch Howe, Jr., University of Georgia

The chromosomes of Bartramidula carolinae. Matti Al-Aish, Duke University

x *Genetic instability in polyploid hybrids between tobacco and its ancestors.* D. U. Gerstel, N. C. State College

Spore germination in Draparnaldia sp. Elton C. Cocke, Wake Forest College

x *On the nature of the light requirement for zygospore germination in* Chlamydomonas reinhardi. Max Hommersand, U.N.C.

x *A survey of the wood anatomy of* Delopyrum, Dentoceras, Polygonella, *and* Thysanella (*Polygonaceae*). James H. Horton, U.N.C.

x *Studies on plant proteins.* T. A. Smith, Duke University

x *Studies on the mechanism of action of amino triazole.* A. W. Naylor, Mason Carter, and David Deamer, Duke University

Relationship between water absorption and salt uptake. William Lopushinsky, Duke University

x *A sexual aquatic Phycomycete parasitic in* Hyalotheca. William J. Koch, U.N.C.

GEOLOGY SECTION

x *Beryl in North Carolina.* Jasper L. Stuckey, State Geologist

x *Are geologists engineers?* John M. Parker, III, N. C. State College

x *Subsurface stratigraphy and structure in part of the Coastal Plain of North Carolina.* William F. Wilson, Division of Mineral Resources

x *Geologic section across Durham Triassic Basin, N. C.* Dana B. Grannell, N. C. State College

North Carolina earthquakes: 1958 and 1959, with corrections to previous lists. Gerald R. MacCarthy, U.N.C.

Guide to the Tertiary-Quarternary marine fossils of the Carolinas. Richard L. Casanova, Paleontological Research Laboratory

x *A survey of clay minerals in North Carolina soil profiles.* R. J. McCracken, S. B. Weed, and L. A. Nelson, N. C. State College

x *Geomorphic expression of former inlets in the*

Outer Banks of North Carolina. John J. Fisher, U.N.C.

MATHEMATICS SECTION

x *The length of a tangent from the point* $P(x_1, y_1)$ *to a conic.* E. F. Canaday, Meredith College

x *Some connected, locally connected, complete, regular, semi-metric spaces which are not arcwise connected, and a very similar such space which is.* Robert W. Heath, Woman's College, U. N. C.

Advantageous uses of the calculus. T. F. Hickerson, U. N. C.

x *On the synthetic process.* J. W. Lasley, U. N. C.

Another new result on irreducibility of Legendre polynomials. Richard F. McCoart, U. N. C.

x *Permutation ordering and identification.* Peter Shahdan, N. C. State College

x *An application of the method of averaging in the theory of satellite motion.* Raimond A. Struble, N. C. State College

x *A note on indeterminate forms.* George Watson, N. C. State College

x *An infinite set of elementary functions.* Wesley O. Doggett, N. C. State College

x *Cyclicly related differential equations and the circulant.* Philip Barnhard and E. J. Pellicciaro, University of Delaware

PHYSICS SECTION

Invited Papers

The visiting scientists program in colleges. Lawrence Slifkin, U. N. C.

The visiting scientists program in high schools. George P. Williams, Jr., Wake Forest College

Contributed Papers

Design and construction of an analog computer. Charles R. Ellis, U. N. C.

An interesting effect of internal print-out in a strained silver chloride crystal. William T. K. Johnson, U. N. C.

Diffusion of silver and gold tracers in crystals of Au-Ag solid solutions. A. Gardner, U. N. C.

Effect of plastic deformation on the optical absorption edge in Ag crystals. M. Miller, U. N. C.

x *Design of a millimeter and submillimeter-wave MASER.* Walter Gordy and Monroe Cowan, Duke University

x *Use of the North Carolina State College subcritical reactor in laboratory teaching.* David H. Martin, N. C. State College

x *Measurement of the transfer function of the Raleigh Research Reactor IV and its control system.* Claude G. Poncelet and Wesley O. Doggett, N. C. State College

x *The variance and fractional standard deviation for the correlation method of measuring nuclear coincidences.* Robert Kennel, Wesley O. Doggett and Linus K. Han, N. C. State College

x *Neutron flux depression in the fuel region of heterogeneous cells.* Charles Hutchins and Wesley O. Doggett, N. C. State College

ZOOLOGY SECTION

x *Occurrence of* Trichinella spiralis *in humans in Piedmont North Carolina, a preliminary report.* Daniel E. Kirk, Catawba College

Winter records of sexual condition of several species of crabs from Grand Terre, Louisiana. Elinor H. Behre, Louisiana State University

A multinucleated amoeba parasite on Hydra littoralis. H. Grady Britt, Wake Forest College

x *Observations on the relative abundance of two commensal crabs of* Chaetopterus. I. E. Gray, Duke University

x *Effects of gamma irradiation on* Palaemonetes and Uca. George H. Rees, U. S. Biological Laboratory, Beaufort

x *The effects of thiouracil and thiourea on the development and degeneration of the mesonephros in the chick.* Charles E. McCreight, Wake Forest College, Bowman Gray School of Medicine

x *The feeding mechanism of fiddler crabs, with ecological consideration of feeding adaptations.* Don C. Miller, Duke University

x *Relationship between oxygen consumption and photoperiodically induced termination of diapause in nymphs of the dragonfly,* Tetragoneuria cynosura. Paul E. Lutz and C. E. Jenner, U. N. C.

Zoogeographical problems of sociological interest today. A. L. Pickens, Charlotte

x *Increased recombination from* Drosophila *females x-rayed before the presumed onset of meiosis.* D. Gale Davis, U. N. C.

x *Some fishes from the White River drainage in Indiana.* L. M. Outten, Mars Hill College

x *Some preliminary histochemical studies of* Haematoloechus *and* Gorgodera, *trematodes of* Rana pipiens. Paul R. Burton, U. N. C.

HIGH SCHOOL TEACHING SECTION

Biology for the talented. Dr. Victor M. Cutter, Jr., Woman's College, U. N. C.

Earth science. Dr. James Batten. U. N. C.

The following two papers were presented by contestants in the State Science Talent Search. Joseph P. Straley was a National Westinghouse Talent Search winner.

The chemistry of non-aqueous solutions. Joseph P. Straley, Chapel Hill High School

The relationship of arterial blood pressure to spontaneous atherosclerosis. Charlene E. Markunus, Reynolds High School, Winston-Salem

ABSTRACTS

Early incorporation of 1-methionine-methyl-C^{14} and dl-methionine-2-C^{14} and radioactive compounds in tissue fractions of adult rats. CECILE H. EDWARDS, EVELYN L. GADSDEN, GERALD A. EDWARDS AND EVANS BOOKER. Adult rats of Wistar strain were fed 1-methionine-methyl-C^{14} (R 12, sacrificed at 20 minutes; R 13, 1440 minutes) or dl-methionine-2-C^{14} (R 11 sacrificed at 16 minutes) by stomach tube following a 10 hour fast. The specific activities of fresh tissues and their water-soluble, fat-soluble, and protein fractions were determined. Of the 18 water-soluble fractions of R 11, pancreas was most active, followed by adrenals, thymus, kidney and liver in decreasing order; whereas, liver was most active in R 13, followed by adrenals, kidney, pancreas and spleen. The fat-soluble fraction of the liver of R 12 was 40 times more active than the next most active tissues of R 11 and R 12. All protein fractions were inactive in R 12, while liver, pancreas, spleen and kidney, in decreasing order, showed activity in R 11. The pattern of radioactivity had changed considerably at 1440 minutes, with bone marrow, adrenal and pancreas containing the most active water-soluble, fat-soluble and protein components, respectively.

The tissue fractions were chromatographed two-dimensionally using phenol and butanol: propionic acid:water, and autoradiograms were prepared. Radioactive compounds in tissue fractions were identified on the basis of R_f values in the two solvent systems and co-chromatography. (Supported in part by a grant, A-1464, from the National Institute of Health, Public Health Service).

Appearance of radiocarbon from 1-methionine-methyl-C^{14} and dl-methionine-2-C^{14} in heme, globin, urine, expired carbon dioxide, tissues and tissue proteins at selected time intervals. EVELYN L. GADSDEN, CECILE H. EDWARDS, GERALD A. EDWARDS AND EVANS BOOKER. Adult, male rats of Wistar strain which had been fasted for 10 hours were fed aqueous solutions of either l-methionine-methyl-C^{14} or dl-methionine-2-C^{14}. Blood samples were taken, either from the aorta by cannula, or from the tail at selected time intervals. Simultaneous and intermittent collection of urine, expired carbon dioxide, and blood was made possible by use of a special metabolism apparatus and procedures previously reported by us. At the end of the experimental periods, the animals were sacrificed. Tissues were removed quickly. Uniform portions were counted with a Geiger-Mueller tube; protein was precipitated from tissue suspensions by use of trichloroacetic acid; heme and globin were removed from red cells; plasma proteins were separated by paper electrophoresis into fibrinogen, *alpha* 1, *alpha* 2, *alpha* 3, *beta* and *gamma* globulins and albumin; $C^{14}O_2$ was precipitated as barium carbonate.

The specific activities of heme, globin, plasma proteins, expired carbon dioxide, tissues and tissue proteins over a period of several selected time intervals will be presented. The relationships between the appearance of the *alpha*-carbon versus the methyl carbon of methionine in these samples will be discussed. (Supported in part by a grant, A-1464, from the National Institutes of Health, Public Health Service.)

Formation of phosphatidyl-dimethylethanolamine in liver preparations. CAMILLO ARTOM AND HUGH B. LOFLAND. Dimethylethanolamine (DME) is a likely precursor in the biosynthesis of choline, or choline-containing phospholipids. It was previously found that liver preparations readily incorporate C^{14}-labeled DME into phospholipids. After incubation with liver slices, the isotopic carbon was present in both DME and choline moieties of phospholipids (M. Crowder and C. Artom, Fed. Proc., **11:** 199, 1952). These findings have now been confirmed and extended. As in previous experiments, after hydrolysis of the lipids, DME was separated by steam distillation (C. Artom and M. Crowder, Fed. Proc., **8:** 180, 1949) and choline precipitated as the Reineckate from the distillation residue. Homogenates, mitochondria, microsomes, and supernates are all active, the greatest activity being in the microsomal fraction. Addition of cytidine mono-, di-, or triphosphate generally enhances the activity of the preparations. Partial separation of phosphatidyl-ethanol-

amine and phosphatidyl-DME was obtained by elution from a silicic acid column with a 4:1 chloroform methanol mixture, whereas phosphatidyl-choline was eluted only by a 3:2 mixture. After mild hydrolysis of the fraction containing phosphatidyl-DME, the water-soluble products were chromatographed on paper and developed with 5% ammonia in ethanol. The data indicated that most of the radioactivity was present as a compound believed to be glyceryl-phosphoryl-DME, which, on further hydrolysis, yielded free isotopic DME. C^{14}-phosphatidyl choline synthesized from labeled DME was characterized by a similar procedure.

Incorporation of C^{14} into phosphatidyl-choline could be explained by methylation of DME to choline which is then incorporated into a phospholipid; or by incorporation of DME into a phospholipid, followed by methylation of its DME moiety. To obtain some evidence on this point, advantage was taken of the inhibition of the incorporation of labeled DME by non-labeled DME, or non-labeled choline. Experiments in which after an initial period of incubation with labeled DME, relatively large amounts of non-labeled DME or choline were added, suggest that, especially in the presence of added methionine, some phosphatidyl-choline is formed by transfer of the methyl group of methionine to the DME moiety of phosphatidyl DME. This concept would be in line with the results of recent experiments on intact rats receiving labeled methionine (J. Bremer and D. M. Greenberg, Biochem. Biophys. Acta, **35:** 287, 1959). (Aided by AEC Contract No. AT-(40-1)-1638 and U.S.P.H. Grant No. A-754.)

Early donor-host considerations in radiation protection by tissue transplants. I. Serotonin level in bone marrow and fetal liver transplants as a factor in radiation protection. GLADYS W. ROYAL AND GEORGE C. ROYAL, JR. Serotonin, naturally occurring in animal tissues, is capable of disrupting cellular integrity when applied locally to the surface of living skin. The question was posed as to whether tissue transplantation as practiced in irradiation studies gave, at the same time, levels of serotonin adequate for injury. Samples of rat bone marrow, mouse bone marrow, and fetal mouse liver were analyzed spectrophotometrically after extraction. Results showed that mouse and rat bone marrow contain detectable amounts of serotonin in the usual dose injected; however, the level in rat bone marrow (40 micrograms per injection) was adequate to produce immediate shock in the host $(B \times D)F_1$ and $(101 \times C_3H)F_1$ mice when the commercial substance was injected in equivalent amounts. Significantly, pooled samples of as many as five fetal livers from LaF_2 or $101 \times C_3H$ mice did not contain detectable amounts of the substance. These findings are suggestive that serotonin level in the transplantation media *per se* is important in radiation protection, since rat bone marrow is often unsuccessful whereas mouse bone marrow and fetal liver are highly successful in mice. (This work was supported in part by a grant from the U. S. Atomic Energy Commission under Contract No. AT-(40-1)-2399.)

Early donor-host considerations in radiation protection by tissue transplants. II. Formation of hemagglutinins in tissue homogenates from irradiated mice after heterologous bone marrow transplantation. GEORGE C. ROYAL, JR. AND GLADYS W. ROYAL. This study was designed to determine whether skin-fixing antibodies can be formed in irradiated mice $(B \times D)F_1$ following rat bone marrow treatments. Animals were irradiated and given rat bone marrow immediately thereafter. Supralethal levels of x-irradiation (900r) were employed throughout. Sacrifices were made after 24, 48, and 72 hours. Lymph node and spleen homogenates as well as serum samples were analyzed for hemagglutinins. Formalinized rat or mouse erythrocytes were utilized in a modification of Boyden's tannic acid technique for the detection of antibodies. Results from the several time studies involving 75 mice showed that spleen homogenates alone contained hemagglutinins to rat globulin. They were not detected until the 48 hour post-irradiation interval and were subsiding by 72 hours. The data suggests that the antibody detected in these experiments is probably univalent, although its skin-fixing property is yet to be characterized. (This work was supported in part by a grant from the U. S. Atomic Energy Commission under Contract No. AT-(40-1)-2399.)

Therapeutic studies on spontaneous atherosclerosis in pigeons. H. B. LOFLAND AND T. B. CLARKSON. White Carneau pigeons were shown to develop spontaneous atherosclerotic lesions which are remarkably similar to those seen in human atherosclerosis (Clarkson, *et. al.*, AMA Arch. Path. **68:** 143, 1959). These findings suggested that the pigeon might be an excellent

experimental animal for research on this disease. In the present experiments, five dietary or therapeutic regimes were studied for their ability to cause regression of these atherosclerotic lesions. Groups of 17 seven-year-old birds were maintained for one year on diets containing, respectively, safflower oil, 10%; Saff-Improved (Abbott), 10%; Crisco, 10%; and no supplement (Controls); or vanadyl sulfate, 0.04%, or diethanolamine, 0.04%, in drinking water. A randomly-selected sample of birds sacrificed initially showed 100% prevalence of lesions, and an average atherosclerotic index (% of surface area of aorta involved in plaques) of 13.7. After one year on the diets, the birds receiving safflower oil and Saff-Improved showed significantly lower atherosclerotic indices (8.9, 9.9, respectively; controls, 13.4). Significant decreases in aorta total lipids and aorta cholesterol values were also noted in the two groups of birds. Atherosclerotic indices and aorta lipid values in birds receiving vanadium, diethanolamine or Crisco resembled those of the controls. Gross and microscopic observations on the atherosclerotic lesions will be summarized.

Non-dynamic state in mammalian tissues. JOHN ERIC WILSON. Tissues of rats were labelled with tritium by the administration of tritiated drinking water (5 μc H^3/ml.) to the mothers and young (cf. Thompson and Ballou, 1956. J. Biol. Chem. **223:** 795). Tritium was removed from drinking water when the litters were 10, 50 or 100 days old. Animals in each litter were sacrificed at intervals and various tissues assayed for tritium, after they had been separated into water-soluble, lipid-soluble and residual fractions. Assays showed that the residual fraction of liver retained about 2% of its total original activity 200 days after tritium withdrawal. In the livers of 50 and 100 day old rats, only about 1% of the lipid-soluble tritium was retained for 200 days, but nearly 20% of the label in 10 day old rats was retained. In brain, 8–12% of the total initial tritium in the residual fraction was retained for 200 days, but 25–50% of the tritium of the lipid-soluble fraction was retained. There was about a 12-fold increase in this metabolically stable, lipid-soluble tritium of brain between the 10th and 50th days of life (during which there is rapid myelination). In contrast to brain, the residual fraction of muscle showed greater retention of tritium (about 20% for the 50 and 100 day old litters) than did the lipid-soluble fraction (about 4–7% retention). Nearly 60% of the tritium of muscle residual fraction of 10 day old animals was retained for 200 days. Results for heart lipids were extremely erratic. There is a laying down of non-dynamic material in the kidney between the 10th and 50th days, and little thereafter. Specific activities in lipid-soluble fractions were consistently higher than those of residual fractions of tissues at the ends of the labelling periods, when tritium was withdrawn.

Some studies on the mechanical aspects of panting. EUGENE C. CRAWFORD, JR. In previous studies on the thoraco-abdominal system the condition of resonance has been looked upon as an abnormal situation resulting from experimental procedure. It is of interest to study respiratory resonance as a naturally occurring phenomenon. Vibratory motion of the thorax of anesthetized dogs mechanically ventilated at a frequency of five or six cycles per second shows a striking similarity to the respiratory movements of a panting dog. This observation led to a series of experiments to determine whether dogs pant at the natural frequency of the respiratory system.

Normal unanesthetized dogs were caused to pant by the warming effect of four 100 watt lamps. Panting frequencies were measured by a Statham Accelerometer attached to the lower lateral rib cage. The mean panting frequency of ten dogs was 5.33 $\pm$ 0.7. The natural frequency of the animals was then determined by the chamber method of Hull and Long (Fed. Proc. **18:** 72, March, 1959), the mean being 5.28 cps $\pm$ 0.3.

One may estimate the work of breathing by the equation of Otis, Fenn and Rahn (J. Appl. Physiol. **2:** 592, 1950). If the heat generated by the panting act is added to the basic metabolic heat of the animal it can be shown by simple calculations that a dog in an environment of 35°C and 50 per cent humidity would have difficulty evaporating enough water to produce the required cooling. The problem confronting the animal is, of course, to provide sufficient ventilation with least effort. Since respiratory impedance is least at resonance, volume-flow is obtained with least effort. This flow at any frequency other than the natural frequency of the system would require greater force to be developed by the respiratory muscles. This would result in the imposition of an additional heat load upon the animal. It is believed that panting dogs make use of the phenomenon of resonance in order to produce a cooling system as economically as possible.

Experiments to prove the foregoing hypothesis have been designed. Measurement of the phase angle between volume-flow and intrapleural pressure in a normal panting dog is planned. If the phase angle is zero, the panting dog must be in "resonance".

Effects of anti-inflammatory drug in canine distemper. N. PATI, Distemper is an infectious disease affecting the nervous system and portal system in dogs. Since distemper is mostly complicated with hepatitis and the use of antibodies was limited for reducing inflammation in the system, an attempt was made to study the effects of the anti-inflammatory drug but-a-zolidine in the treatment of canine distemper. Diagnosis was made on the basis of serological and chemical tests. But-a-zolidine at the rate of 100 mg. per 25 lb. body weight was found to be the most effective to reduce high constant temperature as well as fluctuation in temperature. But-a-zolidine was found to be very satisfactory with vitamin B_1, and the results were not satisfactory in vitamin B_1 itself. But-a-zolidine can reduce the temperature very rapidly. Out of 15 treated dogs, the recovery was very satisfactory in 12. Cortisone preparations with antibodies were failures in these cases.

Trypsin and blood clotting. JOHN H. FERGUSON, S. G. IATRIDIS, ELLA GRAY WILSON, AND H. A. RIERSON. Between 1939 and 1959 many publications of the senior author and colleagues (*ref.* Ferguson, J. H. *Lipoids and Blood Platelets.* U. N. Carolina Press, 1960) supported the idea that trypsin aids blood clotting by functioning as a "thromboplastic enzyme," in conjunction with Ca-ions and a lipid (e.g. cephalin, platelets, or weak tissue thromboplastin), causing the conversion of prothrombin to thrombin. In recent years a number of "accessory" factors have been discovered to play a role in "thromboplastin" and thrombin formation. The time is ripe, therefore, to reinvestigate possible interrelations between these accessory factors and the prothrombin-activating action of trypsin. Experiments with a 2-stage Ca-cephalin-activated "eluate" (prothrombin) system will be presented to show that trypsin needs Ca, cephalin, and "accessory" factors V, VII, and X, but not VIII, IX, or Hageman. In addition, 1-stage tests, namely, plasma recalcification (PCa), plasma-Ca-thromboplastin (PT-Quick's test), plasma-Ca-cephalin (PTT test), and *specific* 1-stage tests for (a) prothrombin, (b) "proconvertin" (VII + X), and (c) AcG (V) will study potentiating effects of added trypsin, with and without added "proconvertin" (VII + X), on normal and certain factor-deficient plasma systems.

Two old specimens of American chestnut in North Carolina. H. BRANCH HOWE, JR. Old specimens of American chestnut, *Castanea dentata*, about 35 miles apart in Yancey and Caldwell Counties, were recently called to the writer's attention.

The Yancey County tree is 6 feet in circumference 3 feet above the ground, subsequently dividing into 2 forks. This tree is approximately 50 years of age as estimated by comparison with a nearby felled chestnut whose circumference and annual rings could be correlated. The Caldwell County tree has a circumference of 53 inches and an estimated age of approximately 37 years.

Both trees are in blighted areas and show signs of previous fungus attack, but the lesions have healed. These trees must have a high degree of blight resistance to have survived to their present age. Consequently, efforts were undertaken to propagate them vegetatively and sexually.

In October and November 1958, cuttings were removed from both trees and stored at 10°C in polyethylene bags containing wet vermiculite. In late March, 1959, additional cuttings were taken from both trees for the writer by Mr. G. G. Miller, Lexington, N. C. In early April 1959, approximately 500 of these fall and spring cuttings were hormone-treated and placed in greenhouse rooting beds. The 4 hormones in the concentrations reported by E. Vieitez (1955. Annales de Edafologia y Fisiologia Vegetal **14:** 483-503) to be most successful in ground-layering Spanish chestnut were applied in lanolin paste. These hormones were (1) 3-indole butyric acid plus 2,4-D (2) 3-indole butyric acid plus 2,4-D-dimethyl amine (3) 3-indole butyric acid plus 2,4-D-triethylamine and (4) 2,4-dichlorophenoxy-n-butyric acid. The writer expresses appreciation to Dr. P. J. Hamrick, Jr., Department of Chemistry, Wake Forest College, for synthesizing the latter 3 compounds.

None of the hormones tested resulted in rooting. In late May, 1959, 27 ground-layerings were made on 8 young 3-foot blight-susceptible chestnut sprouts growing near the Wake Forest College campus. The 4 rooting hormones of Vieitez were again tried. Three months later stems showed marked callus formation but no rooting. Attempts by several other workers to propagate American

chestnut vegetatively have proved refractory (A. H. Graves, 1957. Report of the Annual Meeting of the Nut Growers Association, pp. 76–86).

In late June, 1959, Dr. R. L. Wyatt, Department of Biology, Wake Forest College, and Mr. G. G. Miller, using the paper bag technique, transferred pollen from the Caldwell to the Yancey County tree. Pollen production in Yancey County occurred too late for the reciprocal cross to be made. After 3 months Mr. Miller recovered 2 burrs from 24 bags in Yancey County which yielded a total of 6 nuts. Further crosses between the few additional old American chestnut trees known to the writer are being planned. The writer would appreciate being notified of the location of still other living specimens of American chestnut five feet or more in circumference for inclusion in the crossing program.

Genetic instability in polyploid hybrids between tobacco and its ancestral relatives. D. U. GERSTEL. (Supported by grant G-4851 of the National Science Foundation). Genetic instability has frequently been reported in hybrids between species of *Nicotiana*. Present evidence indicates that instability can also be found among the derivatives of hybrids between *N. tabacum* and its closest relatives. This was first suspected when amphiploids *N. tabacum* x *tomentosiformis* gave for most loci segregation ratios which were not only smaller than the 5:1 gametic output normally expected but even smaller than the 3.7:1 ratio obtainable only from chromatid segregation. Cytological or mutational change from the dominant to the recessive condition could explain the excess of recessives. Evidence for such instability was found not infrequently. A typical example was an amphiploid from fasciated tobacco and normal *N. tomentosiformis* which had fasciated and normal sectors, and thus had lost in parts of the plant the normal factor from *N. tomentosiformis*. Striking evidence for instability was observed in progenies from amphiploid *N. otophora*, relative of *N. tomentosiformis*, and tobacco. An amphiploid of coral tobacco and *N. otophora* yielded in the testcross with recessive coral tobacco 12 coral and 128 carmine segregants. All but 9 of the carmine plants were variegated carmine-coral; i.e., they were instable for the *N. otophora* color gene. In a similar testcross of amphiploid albino tobacco x *N. otophora* there were albino and green segregants; among 1887 green offspring, 12 were green-white chimaeras—here the chlorophyll factor from *N. otophora* was instable only in a small minority of plants. The relations between these various observations and the extent to which instability is mutational or chromosomal requires investigation. Of particular interest is the question whether the genes of the wild species only are instable, or whether markers from tobacco may equally be affected.

On the nature of the light requirement for zygospore germination in a unicellular green alga, Chlamydomonas reinhardi. MAX HOMMERSAND. Mature zygospores of *Chlamydomonas reinhardi* germinate synchronously in the light in ten to eleven hours after transfer to a new agar plate. It was found that the light requirement for germination is entirely replaced by a combination of the amino acids cysteine and methionine in concentrations as low as 10-4M. Homocysteine-thiolactone and homocystine will substitute for methionine, indicating that the latter is not required as a source of C_1 units. Choline is entirely inactive. The cysteine requirement can be replaced by a number of redox compounds, such as flavin mononucleotide, ascorbic acid, cytochrome C, thioglycollic acid and cystine. Redox compounds that promote germination in the dark are active whether in an oxidized or reduced form. Glutathione and homocysteine inhibit germination at 10-4M and the inhibition is reversed by cysteine at 5 to 10 times the concentration of the inhibitor. This inhibition is not reversed by the other redox compounds.

Experiments with labeled sulfate showed that the label is incorporated into the cysteine and methionine of the protein during germination, and that if one supplies unlabeled cysteine as well as labeled sulfate, the label goes only into the methionine. The treatment with labeled sulfate, and unlabeled methionine gave incorporation only into the cysteine of the protein. In this case, it appears that neither cysteine nor methionine is a precursor of the other amino acid.

An examination of the terminal oxidases of the spores and vegetative cells showed that the respiration of the zygospores is sensitive to carbon monoxide, with the inhibition fully reversed by blue light, and that the respiration rate is unaffected by oxygen tensions down to ½%. The respiration of the vegetative cells, on the other hand, is insensitive to carbon monoxide and the terminal oxidase has a low affinity for oxygen. The transformation from the system in the spores to the one in the v cegetativeells is complete and

takes place in the dark between the 7th and the 8th hour of germination.

A survey of the wood anatomy of Delopyrum, Dentoceras, Polygonella, *and* Thysanella (Polygonaceae). JAMES H. HORTON. These four genera are treated by J. K. Small (*Manual of the Southeastern Flora*, 1933) as closely related ones, with a total of thirteen species—one in *Dentoceras*, two in *Thysanella*, five in *Delopyrum*, and five in *Polygonella*. A sixth *Polygonella* species was named in 1937 by V. L. Cory. Species of this group are found only in North America, centered in the southeast, extending north to southern Canada, south to southern Florida, west to Illinois in the north and Texas in the south.

The wood anatomy of this group was surveyed in an attempt to better understand the phylogeny of the group. Since the phylogeny of the vessel elements of the secondary wood has been carefully investigated, a study of this xylem element was undertaken. Throughout the group it appears that vessel element pits are fully bordered and alternate, and perforations are simple. Other commonly used phylogenetic criteria, eg. end wall angle, angularity, length to width ratio, and so on, are quite variable within a single species. Vessel element length proved to be their most promising feature, but mean lengths of vessel elements from the fourteen species as determined in this study are so close that no phylogenetic inferences can be drawn at the species level. Generally, however, *Dentoceras* and *Polygonella* (woody perennials) have shorter vessel elements than *Delopyrum* (herbaceous annuals). The position of *Thysanella* (woody annuals) is not clear.

Stem structure of plants in the group is unusual. Wood parenchyma seems to be entirely lacking, the xylem being composed almost entirely of vessel elements and fibers. *Dentoceras* and *Polygonella* develop primitive wood rays, especially in late wood. *Delopyrum* and *Thysanella* apparently lack rays. The fibers of most, if not all, of the species are living, non-septate, and accumulate starch in such quantity that the xylem must be considered a storage tissue of prime importance. If these similarities in anatomical features are reflected by other characteristics of these plants, they might better be considered as species of the single genus *Polygonella*.

Studies on plant proteins. T. A. SMITH. Paper electrophoresis in an apparatus designed after that described by Grassman and Hannig (Z. Physiol. Chem., **290:** (1952), 1–27) was used in a qualitative investigation of plant proteins. The tissue was macerated in an equal volume of 0.1 molar di-sodium hydrogen phosphate buffer using a pestle and mortar with a small amount of sand. After centrifugation the protein was applied to a horizontal strip of Whatman 3 MM paper and the separation effected with a potential of 6 volts/cm., in a pH 7.6 buffer (μ = 0.35) for periods of up to 15 hours. The proteins were then detected by staining with bromophenol blue after the method of Jencks *et al.* (Biochem. J., **60:** (1955), 205–215).

Extracts of leaves, stems, roots and cotyledons of light-grown pea seedlings and leaves and stems of dark-grown pea seedlings showed a basically similar pattern consisting of a major rapidly-moving band, followed by a slower band of lower intensity. While the protein bands obtained by electrophoresis at pH 7.6 appeared to be the same in light- and dark-grown seedlings, there were apparent differences in amount of several of the proteins.

The major band in the extract of the light-grown leaves probably corresponded to the fraction I protein observed by Singer *et al.* (J. Biol. Chem., **197:** (1952), 233–239). In confirmation of the observations of Lyttleton and Ts'o (Arch. Biochem. Biophys., **73:** (1958), 120–126) on the subcellular localization of this protein in other plants, it was demonstrated that this fraction is associated with particles sedimented by low centrifugal fields from extracts of light-grown spinach and pea leaves. The association of this protein with easily sedimented subcellular particles was demonstrated also in extracts of the shoots of dark-grown pea seedlings.

Studies on the mechanism of action of aminotriazole in plants. AUBREY W. NAYLOR, MASON C. CARTER, AND DAVID DEAMER. Note is taken of the structural resemblance of aminotriazole to histidine and to pyrolles found in such porphyrins as catalase, cytochrome c, cytochrome oxidase, heme and chlorophyll. Emphasis in the work reported has been placed upon the possibility that aminotriazole (ATA) acts as an anti-metabolite in the synthesis of pyrroles and of histidine. Also, since ATA acts as a fairly powerful chelator of iron, investigations have been made to see if ATA's toxic effects are exerted primarily through its iron sequestering ability.

Results with radioactive ATA make it clear

that beans can rapidly metabolize ATA and that ATA does not accumulate as such in stem tips following application to leaves. One of the 14 derivatives accumulates to a marked degree and is ninhydrin sensitive. Glycine and serine both readily supply their carbon skeletons to the derivative. The iron chelate of ATA is still able to induce typical ATA symptoms. It is concluded that under many circumstances ATA seems to prevent formation or utilization of pyrrole rings required in porphyrin synthesis. Evidence is also presented that ATA interferes with histidine utilization or possibly its formation. Because of interference in glycine, serine, and histidine utilization, there is doubtless interference in protein synthesis. One reflection of this is failure to develop chloroplasts following treatment with ATA.

A sexual aquatic Phycomycete parasitic in Hyalotheca. WILLIAM J. KOCH. A new member of the Olpidiaceae is reported as a parasite of the gametangia of the filamentous desmid, *Hyalotheca dissiliens*. It does not grow in the vegetative cells or in the zygotes of its host. The zoosporangial thalli and the zoospores of this parasite resemble those of *Olpidium*, but the sexual mechanism is new to the Olpidiaceae. Within the host protoplast the contents of a small, walled thallus or male gametangium pass through a tube into a larger thallus or female gametangium. This receptive thallus then matures into a spiny-walled resting spore. The mature resting spore, with its adhering empty male cell, resembles somewhat the resting spore of *Olpidiopsis*.

Beryl in North Carolina. JASPER L. STUCKEY. The first reference to beryl in North Carolina, known to the writer, is in Appendix C to Kerr's "Geology of North Carolina," 1875. In that year (1875) J. A. D. Stephenson of Statesville sponsored systematic search that led to the discovery of several localities including one at Hiddenite which produced several splendid emeralds and other types of gem beryl. One emerald 8½ inches long and weighing nearly 9 ounces produced at Hiddenite about 1885, went into the Morgan-Berment collection at New York City. In 1894 a locality was discovered on Big Crabtree Mountain, 4 miles southwest of Spruce Pine, Mitchell County, that produced emeralds and other varieties of beryl. Following this, a number of localities that produced gem quality beryl were discovered in Mitchell and Yancey counties. A third locality where emeralds and other gem varieties of beryl were produced was discovered 4 miles southwest of Shelby, Cleveland County in 1909.

Prior to World War I, the only use for beryl was as a gem stone. About 1920, German and American scientists began experimenting with beryllium and presently that metal is in considerable demand. As a result of the increased demands for beryllium, there has been considerable prospecting for beryl in North Carolina during the past 40 years. No large deposits have been discovered but approximately 65 beryl localities have been found in 21 counties in the Piedmont and Mountain sections of the State. The largest deposits are in the Kings Mountain Spodumene Belt of Cleveland, Gaston and Lincoln Counties where pegmatite dikes contain an estimated 823,000 tons of beryl in rocks containing 0.5 percent BeO.

Are geologists engineers? J. M. PARKER, III. The question posed in the title is largely an exercise in semantics. Definitions of engineering functions that are accepted by the recognized professional engineers stress analysis and design in the solution of problems concerned with the material welfare of mankind under economical conditions. The work of practising geologists employed by industrial concerns commonly falls within the range of engineering as defined. Ordinarily this work does not result in a material product but instead is advisory. This difference probably accounts for existing controversy as to whether geologists are scientists or engineers, and as to the best program for training them.

Subsurface stratigraphy and structure in part of the Coastal Plain of North Carolina. WILLIAM F. WILSON. Information compiled from selected well logs in the Coastal Plain of North Carolina indicates that the Beaufort formation is restricted to the northeastern section of the Coastal Plain. The subsurface extent of the formation is limited in area to counties no further south than Beaufort and extending eastward just south of Cape Hatteras, and no further west than Martin, Bertie, Hertford and Gates. Therefore, during Paleocene time, a basin of considerable, but undetermined size must have existed. A contemporaneous barrier located to the south restricted the deposition of these Paleocene sediments to the northeastern part of the Coastal Plain.

A geologic section across the Durham Triassic Basin, North Carolina. DANA B. GRANNELL. A portion of the Durham Triassic Basin, lying between the Raleigh-Durham Airport and Chapel Hill, North Carolina has been investigated. The area mapped is two miles wide and extends in a west-northwest direction for a distance of approximately fifteen miles.

The fault-produced basin contains sediments derived from source areas both to the east and west, laid down in lens and wedge shaped beds. The sediments are closely jointed and probably faulted, the faulting being obscured by weathering. The sediments may be divided into six stratigraphic units, the first being at the southeastern end of the area and the sixth being at the northwestern end. Unit one is 1½ miles in outcrop width and consists mainly of yellow-brown to red sandstone, siltstone, and conglomerate. The conglomerate, the distinctive rock of the unit, contains rounded fragments (mainly quartz) up to one foot in length. Unit two, 3½ miles in outcrop width, contains red, possibly arkosic sandstone in beds up to three feet in thickness, strikingly lenticular in form. Unit three, approximately 150 yards wide, is characterized by alternating layers of red to green to buff shales and limestone with associated chert. Unit four, 3¼ miles in outcrop width, is similar to unit two except that it is more micaceous and contains more shale. Unit five contains alternating layers of red micaceous shale and yellow, coarse, crossbedded, poorly sorted sandstone. Unit six is similar to unit one.

Economically, the area has little to offer. Coal which is present in a geologically similar environment to the south is missing here. The limestone is present in too small quantities to be of interest.

A survey of clay minerals in North Carolina soil profiles. R. J. MCCRACKEN, S. B. WEED AND L. A. NELSON. Clay mineral content of some representative North Carolina soil profiles have been summarized, the estimates being based primarily on results of X-ray diffraction studies of soil clays, complemented in varying extent by cation exchange capacity determinations, differential thermal analyses and electron micrography. This summary is based on research in progress by us, as well as previous work (McCaleb, S. B., manuscript for N. C. Agricultural Experiment Station Bulletin; Nyun, M. A. and S. B. McCaleb, Soil Sci. **80:** 27–41, 1955; Southern Regional Bulletin 61, Project S-14 published by Va. Agricultural Experiment Station, Blacksburg.)

Major clay minerals present are kaolinite and an aluminum-interlayered triphormic mineral (or mineral group). This latter species (or group) has a basal spacing of approximately 14.3 Angstroms before heating and exhibits varying degrees of thermal stability of the basal spacing when heated. It exhibits varying expandability when glycolated following chemical treatment to remove interlayer Al. This Al-interlayered mineral or group occurs in largest amounts in A horizons of North Carolina soils.

Illite, or clay mica, is present in minor amounts in most soils of the Mountains region and a few soils of the Piedmont. Gibbsite is present in minor amounts in most profiles studied, including well-drained soils of the Coastal Plain. This mineral is present in major amounts in certain soil profiles of the foothills and intermountain basins (Cate, unpublished MS thesis, N. C. State College). Montmorillonite is present as a major component of the lower portion of the B and C horizons of a few soils of the Piedmont; these comprise about five percent of the total area of the Piedmont region.

Geomorphic expression of former inlets in the outer banks of North Carolina. JOHN J. FISHER. Inlets are a much more common feature of barrier beaches than is often assumed. There are presently only four major inlets open along the North Carolina Outer Banks between Cape Henry and Bogue Inlet. But reference to historical documents and coast maps show that there have been more than twenty different inlets within the last 350 years and as many as eight open at the same time. This number together with an inherent migrating nature shows the ubiquity of inlets.

Geomorphic expression of present and former inlets is found in lagoon, ocean and beach environments. Lagoon bottom topography shows tidal surge channels of the former inlets. Greater width of the barrier beach in these areas is ascribed to former flood tidal delta deposition. Beach front material, eroded by wave action and transported by long-shore and tidal currents, finds final deposition, in a large part, as flood tidal deposition behind the barrier beach from which it was eroded. Thus landward retrograding of a barrier bar often assigned to dune migration activity may be accomplished in part by flood tidal delta deposition.

The ocean is affected only to the extent that

the offshore profile reflects the former ebb tidal deltas and at times gives rise to shoal areas extending perpendicular to the present coast.

The barrier beach may be an area of low barren sand but it is more often dune covered. The lateral migration of an inlet, as well as subsequent closure, produces areas of beach not conducive to vegetation and thus susceptible to dune formation.

The length of a tangent from the point $P(x_1, y_1)$ to a Conic. E. F. CANADAY.

I. The Ellipse $b^2x^2 + a^2y^2 - a^2b^2 = 0$.

The tangents from point P touch the ellipse at the points of intersection of the ellipse with the chord of contact $b^2x_1x + a^2y_1y - a^2b^2 = 0$. The points of intersection are given by the equations;

$$x = \frac{a^2b^2x_1 \pm a^2y_1\sqrt{b^2x_1^2 + a^2y_1^2 - a^2b^2}}{b^2x_1^2 - a^2y_1^2}$$

$$y = \frac{a^2b^2y_1 \mp b^2x_1\sqrt{b^2x_1^2 + a^2y_1^2 - a^2b^2}}{b^2x_1^2 + a^2y_1^2}$$

The lengths of the tangents are the distances from these two points to $P(x_1,y_1)$ After a fairly long process of simplification this length of tangent t reduces to

$$t = \frac{\sqrt{b^2x_1^2 + a^2y_1^2 - a^2b^2}}{b^2x_1^2 + a^2y_1^2}$$

$$\sqrt{(b^4x_1^2 + a^4y_1^2} \pm 2x_1y_1(a^2 - b^2)\sqrt{b^2x_1^2 + a^2y_1^2 - a^2b^2} + (x_1^2 + y_1^2)(b^2x_1^2 + a^2y_1^2 - a^2b^2)$$

Since the lengths of the two tangents differ only because of the $\pm$ sign under the radical we have the following property (a) and others which would be expected.

(a) The tangents to the ellipse from point P are equal *if and only if* point P lies on one of the axes of the ellipse (making the term $2x_1y_1$ $(a^2 - b^2)$ equal to zero.

(b) If the point is on the ellipse $t = 0$

(c) If the point P is within the ellipse the length t is imaginary.

(d) If $a = b = r$ the ellipse of course becomes a circle and the t reduces to $t = \sqrt{x_1^2 + y_1^2 - r^2}$ if the center is at the origin and

$$\sqrt{(x_1 - h)^2 + (y_1 - k)^2 - r^2}$$

if center is (hk).

(e) The lengths of tangents from $P(x_1,y_1)$ to the ellipse $(x - h)^2/a^2 + (y - k)^2/b^2 = 1$ may be found by replacing x_1 by $(x_1 - h)$ and y_1 by $(y_1 - k)$ in the above formula for the length of t.

II. The Hyperbola $b^2x^2 - a^2y^2 - a^2b^2 = 0$

Replace b^2 by $-b^2$ in the above formula for the length of t and we have the lengths of tangents to the hyperbola. Properties corresponding to a,b,c,e above hold for the hyperbola. If point P is on the asymptote the length of the tangent involves division by zero.

III. The Parabola $y^2 = 4\,px$.

The chord of contact is $y_1y = 2\,p\,(x + x_1)$.

The intersections of the parabola and the chord are

$$x = \frac{(y_1^2 - 2px_1) \pm y_1\sqrt{y_1^2 - 4px_1}}{2P};$$

$$y = y_1 \pm \sqrt{y_1^2 - 4px_1}.$$

The length of the tangents from $P(x_1,y_1)$ to these points are

$$t = \frac{\sqrt{y_1^2 - 4px_1}}{2p}\sqrt{(\sqrt{y_1^2 - 4px_1} \pm y_1^2) + 4p^2}.$$

(a) The tangents are equal if and only if the point P is on the axis of the parabola that is so $y_1 = 0$ following the $\pm$ sign in the formula.

(b) If p is on the parabola $t = 0$.

(c) If p is on the focus side of the parabola then t is imaginary.

(d) Tangents to $(y - k)^2 = 4\,p\,(x - h)$ may be found by replacing x_1 and y_1 by $x_1 - h$ and $y_1 - k$ respectively.

(e) It is interesting to note that if the points of tangency be denoted by (x_2,y_2) and (x_3,y_3) then $x_2x_3 = x_1,y_2 + y_3 = 2y$, or $x = \pm\sqrt{x_2x_3}$, $y_1 = (y_2 + y_3)/2$.

Two connected, locally connected, complete, regular, semi-metric spaces which are not arc-wise connected, and a very similar such space which is. ROBERT W. HEATH. In *Foundations of Point Set Theory* R.L. Moore proved that a locally connected complete Moore space is arc-wise connected. Many of the theorems that hold in a complete Moore space also hold in a Cauchy complete, regular semi-metric space. In this paper we give two examples of Cauchy complete regular semi-metric spaces in which the arc theorem does not hold, even though one of the two spaces is compactly connected. We also give an

example of a very similar semi-metric space which is not a Moore space, but which is arc-wise connected.

On the synthetic process. J.W. LASLEY, JR. The usual application of the synthetic process, whether to division, substitution, or power series employs a linear divisor with the leading coefficient unity.

This paper calls attention to the fact that the leading coefficient need not be unity, nor the divisor indeed be linear. The process may be so modified as to divide any polynomial by another of no higher degree. It can be adapted also to solve the general problem of expressing a polynomial as a power series of any other polynomial, provided the latter polynomial is not of higher degree.

Permutation ordering and identification. PETER SHAHDAN. A general method of ordering all the permutations of n objects, some of which may be alike, is devised; a theorem is proved partitioning the total number of permutations of the set into sub-sets, whence it becomes possible to

1. Identify the cardinal number of any given permutation;
2. Obtain any required permutation when given its cardinal number.

An application of the method of averaging in the theory of satellite motion. RAIMOND A. STRUBLE. A method of averaging reminiscent of the method of Krylov-Bogoliubov-Mitropolski, is employed in a study of the motion of a satellite of an oblate spheroid. The reduced nonlinear equations are integrated in closed form and the complete solution is expressed in terms of standard elliptic integrals. This solution reveals the interesting secular effects of the oblateness of the spheroid without recourse to perturbational procedures.

A note on indeterminate forms. GEORGE C. WATSON. It has been noted that many textbooks prepared for the teaching of sophomore calculus present the indeterminate form 0^0 by examples exclusively of the type $\lim_{x\to 0} x^x$ and $\lim_{x\to 0} (\sin x)^{\tan x}$, which invariably yield a limit of 1. The good student is inclined to ask, if such is always the case, why is it necessary to say that $a^0 = 1$, provided that $a \neq 0$. This has led to the construction of a simple example, and the proof of a more general theorem.

THEOREM: If $f(x)$ is continuous and of the order of x at the origin (i. e., $\lim_{x\to 0} \frac{f(x)}{x} = b$, $0 < b < \infty$), if $g(x)$ is continuous at the origin, and if $h(x)$ is of the order of $\ln x$ in a right hand neighborhood of the origin (i. e., $\lim_{x\to 0+} \frac{h(x)}{\ln x} = c$, $0 < c < \infty$), then

$$\lim_{x\to 0+} [f(x)]^{g(x)/h(x)} = \exp \frac{g(0)}{c}$$

An example of this more general theorem is then given.

An infinite set of elementary functions. WESLEY O. DOGGETT. The behavior of a set of functions defined by simple power series was investigated. Reversions relations, asymptotic forms, periods, derivatives and integrals were developed. Applications in Mechanical, Civil and Electrical Engineering, Physics and Statistics are given. Experimental confirmation of theoretical work in nuclear physics invoking these functions is presented.

Cyclicly related differential equations and the circulant. PHILIP BARNHARD AND E.J. PELLICCIARO This paper concerns itself with the system of differential equations

$$(1) \qquad y_i' = \sum_{k=1}^{n} A_k y_{i+m+hk}, \quad (i = 1, \cdots, n;\ y_{j+n} \equiv y_j),$$

where h and m are integers and the coefficients A_k are integrable functions on $[a,b]$. W. M. Whyburn [A set of cyclicly related functional equations. Bull. Amer. Math. Soc. **36:** 863-868. 1930] exhibited the exponential nature of the solutions of (1) by obtaining directly, with the aid of the Lagrange resolvent and its properties, the general solution of the equations. The present paper uses the theory of the circulant to solve (1). This is done by showing that (1) is equivalent to the matrix differential equation $Y' = BY$, where B is a circulant. A non-singular transformation is applied reducing B to canonical form and the resulting system solved. This same method is then applied to solve the more general system

$$y_i' = \sum_{k} A_k y_{i+\alpha(k)}, \quad (i = 1, \cdots, n;\ y_{j+n} \equiv y_j).$$

Design for a millimeter and submillimeter-wave Maser. WALTER GORDY AND MONROE GOWAN. A

new proposal for the amplification and generation of millimeter and submillimeter wave radiation by means of a molecular system will be outlined. The device utilizes selective Stark components of a symmetric-top molecule to achieve Maser action.

Use of the North Carolina State College sub-critical reactor in laboratory teaching. D. H. MARTIN. The present uses of the sub-critical assembly at N. C. State College consist of two laboratory experiments, one consisting of a flux mapping of the lattice and the other being an exponential experiment. These experiments are described, together with measurement techniques using copper cylinders as detecters. Typical results are given, showing a thermal flux peak in the reflector for the first experiment and giving a measured value of 26×10^{-4} cm^{-2} for the material buckling in the second case.

Measurement of the transfer function of the Raleigh Research Reactor IV and its control system. CLAUDE G. PONCELET AND WESLEY O. DOGGETT. A series of frequency response measurements was made to obtain the transfer function of the homogeneous North Carolina State College Reactor IV. A sinusoidal reactivity input was obtained by oscillating the control rod with an electrical signal from a low frequency oscillator over the frequency range 0.02–2cps. Observations of the rod and power oscillations were made with a brush recorder. The gain was measured with a simple potentiometer; the phase shift was measured by adding the input and the output signal in order to obtain a signal proportional to the phase shift. The observed transfer function was in good agreement with the theoretical six-group transfer function derived from the kinetics equations. The magnitude of the neutron lifetime was evaluated from the experimental gain measurements. The transfer function of the automatic control system was determined with the same method. The system was found to be a pure integrator with a simple K/s transfer function. The root-locus technique was then applied to the total control loop of the reactor, which consists of the reactor, neutron chamber, linear amplifier, comparator, and automatic control system. The control system was found to be absolutely stable and to have no oscillations in its response.

Approximate P_3 solution to Boltzmann transport equation in slab and cylindrical fuel regions. CHARLES ROBERT HUTCHINS. An approximate P_3 spherical harmonics solution to the Boltzmann transport equation is developed for a reactor cell fuel region in slab and cylindrical geometries. The approximation is a rapid hand-calculational method which incorporates general anisotropic scattering and includes a procedure for representation of the fuel flux depression as a single function, cosh kz or I_0 (kr) for slab and cylindrical geometries, respectively. The results from the approximation are equivalent to or better than the accuracy of the results from the P_3 solution. Calculations are presented for nine reactor cell configurations.

Transient temperature distributions for a nuclear reactor with exponentially time varying and chapped-line spatial power distribution. ROBERT SHULTZ AND WESLEY O. DOGGETT. Earlier work on heat transfer in a bare reactor with exponential power variation was extended to a reflected reactor. The heat flow from the fuel element to the coolant is described by coupled partial differential equations with dependence on time and the axial distance. Integral functions previously encountered and numerically integrated by other authors were integrated in terms of well known functions.

Theoretical evaluation of the variance for the statistical correlation method for measuring daughter half-lives in the $10^{-4} - 10^{-1}$ second range. ROBERT PHILIP KENNEL. A time correlation method proposed by Petroff and Doggett (Petroff, Michael D. and Doggett, Wesley O. 1956. "Application of a statistical correlation function for determining daughter half-lives in the range 10^{-4} to 10^{-1} second." Rev. Sci. Instr. **27:** 838–843) for investigating two connected nuclear disintegrations has been studied. The method permits determination of half-lives in the range 10^{-4} to 10^{-1} second for daughters in equilibrium with long lived parents without the count rate restrictions imposed on the usual delayed coincidence method by the occurrence of accidental coincidences. The variance of the correlation measurement is theoretically evaluated for a radioactive source cascade with negligible background and is determined for backgroumd activity with no cascade present.

Occurrence of Trichinella spiralis *in humans in Piedmont North Carolina—a preliminary report.* DANIEL E. KIRK. Trichinella spiralis larvae, revealed by the press method and microscopic examination, have been found in six of one hundred human diaphragms. The survey was done on post mortem material, in Salisbury,

North Carolina, during 1958–1960. Sixty-five per cent of the subjects were between the ages of 47 and 78 years. One case was from a terminal illness with all clinical characteristics of acute trichinosis. One other specimen showed moderately heavy infestation and incomplete calcification, suggesting a recent infection.

Results thus far suggest a need for continual public education on trichinosis and a constant alertness by physicians for symptoms of this parasite.

Observations on the relative abundance of two commensal crabs of Chaetopterus. I. E. GRAY. The most common commensals of the marine annelid, *Chaetopterus variopedatus*, are two decapod crustaceans, *Pinnixa chaetopterana* and *Polyonyx macrocheles*. These range in their distribution from southern New England to Brazil. Seldom are Chaetopterus tubes found without one species or the other. During the last half century changes in their relative abundance have been noted. In the Woods Hole region, at the beginning of the century, Pinnixa was common but Polyonyx was rarely observed. In 1913 Pearse found that about 22 per cent of these two crabs were Polyonyx. By 1959 Polyonyx had increased to the extent that the Polyonyx-Pinnixa ratio was 66:34. This marked increase of Polyonyx in the Woods Hole region is perhaps due to amelioration of climate, permitting the crab to extend its range northward.

In the Beaufort, N. C. area the picture is somewhat different. In 1905 Enders found that Polyonyx made up 83 per cent of these crabs. In marked contrast studies in 1958 and 1959, when crabs from Chaetopterus tubes taken from various types of substrate were counted, it was found that Polyonyx amounted to only 39 per cent of the crabs. However, the ratio of Polyonyx to Pinnixa varied on different substrates, with mud yielding 26:74; mud-sand 34:66; and sand-shell 54:46. The change in abundance of Polyonyx in the Beaufort region is hard to explain except on the basis that there is now more mud in the environment than in Enders' time. Commensals taken in February, 1960, from the sandy beaches of Clearwater, Florida, gave a Polyonyx-Pinnixa ratio of 82:18, similar to that found by Enders to exist in the Beaufort region in 1905.

Effects of gamma irradiation on two decapod crustaceans, Palaemonetes pugio *and* Uca pugnax. GEORGE H. REES. Fiddler crabs (*Uca pugnax*) and grass shrimp (*Palaemonetes pugio*) were subjected to various exposures of gamma irradiation from a cobalt-60 source as a part of a program to measure the relative sensitivity of decapod crustaceans to ionizing radiations. Observations were made of the mortality rates, number molting, and, in some instances, hatchability of the eggs and development of the larvae. An interesting finding in these experiments was the relatively low dosages required to affect these invertebrates. *Uca pugnax* exhibited an $LD_{50}/30$ days of 8,000 roentgens and *Palaemonetes pugio* had an $LD_{50}/30$ days of 1,400 roentgens. In the case of *Palaemonetes* the radiosensitivity is close to that of some mammals. One of the effects of the lower dosages in tests with *Uca* was a prolongation of the intermolt period.

The effects of thiouracil and thiourea on the development and degeneration of the mesonephros in the chick. CHARLES E. MCCREIGHT. In the course of their embryological development reptiles, birds, and mammals form in succession three pairs of kidneys. The second of these, the mesonephros, becomes a sizeable, and apparently functional, organ during this period, but it undergoes decided degeneration by the time of birth. The thyroid hormone exerts manifold influences upon processes of development and growth. Inhibition of the thyroid hormone in developing chicks has been utilized in this experiment to arrive at the effects of the hormone on development and degeneration of the mesonephros.

Solutions of thiouracil, thiourea, or saline were injected into the albumen of chick eggs at 6, 12, and 18 days of incubation, and chicks were sacrificed daily from the 12th day of incubation until the time of hatching. After fixation of embryos in Bouin's solution, the thyroid glands and mesonephric kidneys were removed and weighed. Histologic sections were prepared.

The effectiveness of the method of administration of the antithyroid drugs was verified by enlargement of the thyroid glands. Thiourea proved more effective than thiouracil. The dosages employed were effective for about one week. General effects noted included dwarfing of the chicks, larger collections of subcutaneous fat, delay or prevention of withdrawal of the yolk sac and hatching. There was delay in both development and degeneration of the mesonephros in thyroid-inhibited chicks, and in spite of the dwarfing effects of the anti-thyroid drugs, these chicks developed mesonephric kidneys of at

least as large size as those developed by control chicks. Microscopic examination showed far more deterioration of malpighian bodies and tubular epithelia in mesonephroi of control chicks than in those of the thyroid-inhibited chicks. It is concluded that the thyroid hormone pushes both the development and degeneration of the mesonephros.

The feeding mechanism of fiddler crabs, with ecological considerations of feeding adaptations. DON C. MILLER. Three species of fiddler crabs occur at Beaufort, N. C.: *Uca pugilator*, the sand fiddler, and *U. pugnax* and *U. minax*, which inhabit the marshes. Since the character of the substratum appears to influence where the species will live, analysis of the feeding mechanism was undertaken to observe its role in determining distribution.

In feeding, detritus associated with the substratum is scraped up by the crab's minor chela and carried to the buccal cavity where fine food and silt is sorted from the coarse mineral fraction. A flotation process, utilizing water pumped from the gill cavity, aids in freeing food and silt from the heavier coarse material. This is followed by action of the mouth parts, which pass the finer material upward to the mouth and eject the coarser fraction from the buccal cavity.

Uca minax generally feeds on a muddy substratum, and thus rarely takes into the buccal cavity material requiring sorting before ingestion. Accordingly, the hairs covering the mouth parts are elaborated only slightly: woolly hairs are abundant, facilitating accumulation of fine material, and a few second maxilliped hairs are spoon-tipped to aid in the manipulation of heavier matter. *U. pugnax* may inhabit marshes recently built up over sand, and there a small amount of sand would be taken into the buccal cavity during feeding. A corresponding modification of the mouth part hairs is seen in the second maxilliped, with a band of spoon-tipped hairs so placed to facilitate withdrawal of coarse grains from central areas of the buccal cavity. In *U. pugilator*, not only is the entire second maxilliped fringed with closely set spoon-tipped hairs for the same purpose, but also bristles of the first maxilliped endites are broadened or spooned to provide a surface against which sand may be scoured and food material removed. Thus, the modification of the mouth part hairs in *Uca* does appear to be one important factor governing their distribution: the marsh crabs are not adapted to feed on a coarse substratum, while *U. pugilator* appears to prefer to inhabit the protected sand beach where it can obtain food of a relatively high nutritive value.

Relationship between oxygen consumption and photoperiodic induction of the termination of diapause in nymphs of the dragonfly Tetragoneuria cynosura. PAUL E. LUTZ AND C. E. JENNER. Oxygen consumption and photoperiodic induction have been studied concurrently in nymphs of *Tetragoneuria cynosura*. From November, 1959 until mid-April, 1960, seven collections of last-stage nymphs, each of approximately 15 individuals, were made from University Lake near Chapel Hill, N. C. Employing the Warburg respirometer, the rate of oxygen consumption was measured for individual nymphs at 22° C. the day following collection and thereafter at approximately weekly intervals. Half the nymphs of each collection were placed on a 14-hour photoperiod and the remaining nymphs were held on an 11-hour photoperiod.

In an experiment started on November 14, respiratory rates for nymphs of both groups were at first relatively low. After approximately 3 weeks, respiratory rates for the long-day nymphs increased dramatically and continued to do so until emergence (average time to emergence was 34 days). The development for the short-day nymphs was greatly delayed and, on the average, these nymphs emerged almost 2 months later than those on the 14-hour photoperiod. The respiratory rates of the short-day nymphs also showed a marked increase prior to emergence and, once the rate began to increase for a particular individual, the rate curve followed a path similar to that shown by the 14-hour individuals.

In collections made progressively later, less time was required to emergence for both the 14-hour nymphs and also for the 11-hour individuals. Thus the difference in time to emergence for the nymphs on the two photoperiods also became progressively less, and little differences between the responses to the two photoperiods were shown for nymphs collected in March and April.

A high respiratory rate of approximately 80 micro-liters of oxygen consumed/hour/nymph is apparently required before emergence can take place. By measuring oxygen consumption in nymphs collected from nature at frequent intervals in late March and early April, we were able to predict accurately the timing of the first emergences in nature. The present study seems to be

the first in which changes in oxygen consumption rate have been determined in Odonata nymphs from diapause up to emergence.

Increased recombination from Drosophila *female X-rayed before the presumed onset of meiosis.* D. GALE DAVIS. Data from many experiments have led to the logical conclusion that induction of crossing over occurs in the gonial cells of Drosophila. Ovaries of larval *D. melanogaster* females reportedly lack oocytes, and thus constitute a site for the direct treatment of oogonia. Late third instar heterozygous *rucuca* larvae were X-irradiated. Doses approximating 1000r, 2500r, and 4000r were administered. Females were reared, testcrossed, and scored for chromosome-III recombination. Induced recombination was measured in two brood from the first seven days of maternal life. This was detectable both as an over-all and a regionally differentiated effect, with regions nearer the centromere being more affected. Non-crossover classes tended to reflect treatment and age variations in crossing over. These broods were found to be highly heterogeneous ($P < 0.005$, $P = 0.023$); later broods were much more homogeneous. Therefore, oogonia are manifestly competent targets for the X-ray induction of crossing over.

Some fishes from the White River drainage in Indiana. L. M. OUTTEN. From August 15 to 18, 1959, collections of fishes were obtained from Sycamore Creek, a tributary of the West Fork of the White River, at Bradford Woods, located six miles north of Martinsville, Morgan County, Indiana. The following species were present: golden redhorse-*Moxostoma erythrurum* (Rafinesque), hog sucker-*Hypentelium nigricans* (LeSueur), stoneroller minnow-*Campostoma a. anomalum* (Rafinesque), western blacknose dace-*Rhinichthys atratulus meleagris* Agassiz, Northern creek chub-*Semotilus a. atromaculatus* (Mitchill), bluntnose minnow-*Pimephales notatus* (Rafinesque, suckermouth minnow-*Phenacobius mirabilis* (Girard), silverjaw minnow-*Ericymba buccata* Cope, spotfin shiner-*Notropis spilopterus* (Cope), southern sand shiner-*Notropis d. deliciosus* (Girard), central johnny darter-*Etheostoma n. nigrum* Rafinesque, rainbow darter-*Etheostoma caeruleum* Storer, barred fantail-*Etheostoma f. flabellare* Rafinesque, green sunfish-*Lepomis cyanellus* Rafinesque, and northern mottled sculpin (muddler)-*Cottus b. bairdii* Girard. Sincere appreciation is expressed to Professors P. L. Whitaker and Robert W. Tully of Indiana University for suggestions and permission for collecting and to Dr. Edward C. Raney of Cornell University for guidance and encouragement.

Some preliminary histochemical studies of Haematoloechus *and* Gorgodera, *trematodes of* Rana pipiens. PAUL R. BURTON. The McManus paraffin-Sudan Black B technique (1946, J. Path. and Bact. **58:** 93–95) has been used by several workers to demonstrate the distribution of certain fats. According to Bullock (1949, J. Morph. **84:** 201–223), the method is relatively specific for phospholipid. This conclusion is based on the assumption that fixation in a solution of cobalt-formalin, precipitation by dichromate, and acetone dehydration will render phospholipid relatively insoluble in paraffin, xylene, and higher alcohols, while other fatty material will be dissolved.

In sections of a frog lung fluke, *Haematoloechus medioplexus* Stafford, the aforementioned technique demonstrates the following strongly positive regions: (1) epicuticle, (2) Mehlis' gland cells and (3) epithelial cells lining the main excretory canal. Moderately positive regions include the body musculature, some of the parenchymal cells, immature vitelline cells in the follicles, and the epithelial cells lining the intestinal caeca.

Even without treatment with dichromate, sections show some darkly stained regions. In favorable sections the basement membrane separating the cuticle from the underlying musculature can be seen as an intensely stained line. Some parenchymal cells, the connective tissue surrounding the internal structures, Mehlis' gland cells, and caecal cells are moderately to heavily stained without post-chroming. It may be that a different lipid fraction is differentiated by eliminating the dichromate treatment.

Sections of *H. medioplexus* and of a frog bladder fluke, *Gorgodera* sp., were subjected to the Bauer-Feulgen technique for the demonstration of glycogen. Treatment of sections for one hour with a 1% animal diastase solution at 37° C. provided controls. The diastase was dissolved in Niu-Twitty salt solution (pH 7.6–7.8).

Axmann (1947, J. Morph. **80:** 321–334) studied the distribution of glycogen in *H. medioplexus* and *Gorgodera amplicava* Looss, among other trematode species. The present study confirms the observations of Axmann, and the observations mentioned below supplement the original work.

In general, glycogen deposits appear to be extracellular. However, the fully developed or mature spermatozoa of both species are positive for glycogen, but the germinal, gonial, spermatocyte, and spermatid cells from which the sperm arise are negative. The connective tissue surrounding the internal structures contains many small glycogen-positive particles.

Hematoxylin-staining bodies have been reported in the cytoplasm of developed oocytes of numerous trematodes, including *H. medioplexus* (Pennypacker, 1940, J. Morph. **66:** 481–495) and *Gorgoderina attenuata* (Stafford) (Willey and Koulish, 1950, J. Parasit. **36:** 67–75). *Gorgoderina* is closely related to *Gorgodera*, and an examination of the ovarian oocytes of the species of *Gorgodera* studied in this investigation reveals hematoxylin-staining bodies similar to those described for *Gorgoderina attenuata*. In *H. medioplexus* such stainable cytoplasmic material usually appears as one or a few small bodies, while in *Gorgodera* sp. the material forms a single, rather large mass. Inasmuch as several workers have suggested that this cytoplasmic material is nutritive, it is perhaps significant that it is negative for glycogen as demonstrated by the Bauer-Feulgen method.

Abnormal cells with pyknotic nuclei are occasionally observed in the gonads of trematodes. A specimen of *H. medioplexus* possessing abnormal cells in the testes was tested for glycogen. Such cells—and what appeared to be their disintegration products—were found to be strongly positive. Normal testicular cells which were present were negative for glycogen, with the exception of the developed spermatozoa. (Note: Research supported in part by a fellowship from the National Science Foundation.)

PROCEEDINGS OF THE ELISHA MITCHELL SCIENTIFIC SOCIETY
1959–1960

554TH MEETING, OCTOBER 13, 1959

G. R. CANNEFAX (Medicine): *Some Physical and Chemical Properties of Reiter Protein Antigen.*

J. W. CARR III (Computation Center): *Research Use of the Univac 1105 Digital Computer.* The presence of the Univac 1105 digital computer as part of the Computation Center of the Consolidated University of North Carolina, and located at Chapel Hill, allows the faculty or graduate student researcher an opportunity to use one of the largest present digital computers and processors as part of his regular work.

The Univac 1105, in operation since August 1959, was brought to the University through the aid of the National Science Foundation, Bureau of the Census, and Remington-Rand Univac Division of the Sperry-Rand Corporation. At present, time is available for both unsupported and supported research activities, as well as for use by members of formal university courses. Short, intensive courses, training students and faculty members in the use of so-called "automatic programming" techniques, are aimed at making use of the computer available to as wide a set of users as possible.

Present activity has indicated very powerful applications are possible in the case of large statistical analysis, numerical solution of ordinary and partial differential equations, and "symbol manipulational" procedures, ranging from natural language translation to machine aids to mathematical and logical investigations.

Twenty-one regular members and 84 associate members were elected from 13 science departments in the University.

555TH MEETING, NOVEMBER 10, 1959

C. W. GOTTSCHALK (Medicine): *The Countercurrent Hypothesis for Urine Concentration.*

B. S. DEWITT (Physics): *Some Problems in Lunar Exploration.* A resume was given of the surface features of the moon which are presently visible from the earth (e.g., craters, rays, maria, rills, valleys, craterlets, and special mountain systems). The current theories of the origin of these features were then discussed. The disagreements between these theories were emphasized and a program was outlined for resolving them when direct exploration in space becomes a reality. The main important measurements and experiments which should be done include: (1) sampling of the "filler" material of the maria to resolve the dust *vs.* lava controversy; (2) seismic measurements to determine depth of the maria and internal lunar structure; (3) search for primitive biological species; (4) heat flow measurements to study internal energy sources; (5) measurements of surface radioactivity; (6) magnetic field measurements; (7) a general selenographical survey with particular attention to the Imbrium system, the Alpine, Borda and Rheita valleys, the Haemus mountains, the Appenines, the Alps, rills, fissures, escarpments, and the craterlets west of Copernicus; (8) a catalog of crater profiles to study the dynamics of impact explosions.

556TH MEETING, DECEMBER 8, 1959

J. H. WELLS (Mathematics): *Real Number Sequences.*

D. G. HUMM (Zoology): *Some Aspects of Metabolism in the Mushroom* Agaricus campestris.

557TH MEETING, JANUARY 12, 1960

H. HOTELLING (Statistics): *The Behavior of Some Standard Statistical Tests under Non-Standard Conditions.* Statistical confidence intervals for unknown quantities are now usually established after observations, with probabilities that are exact if the observations have normal distributions of certain kinds. The same is true of tests of significance. These procedures are defended on the grounds that both theory and certain investigations of special cases point toward actual distribution of observations, suitably corrected, having approximately the normal form; and further, that mathematical theory indicates that with a sufficiently large number of observations, the distribution of means, moments, and numerous other statistics have nearly the normal form even if the basic observations have distributions rather far from normality.

These defenses are not complete, and especially for small samples, or when very stringent levels of significance are required, the inapplica-

bility of the assumptions sometimes involves really large errors in the probabilities used. Non-parametric methods can then sometimes be applied, but are generally insufficient, discarding much of the information inherent in the observations.

This paper reports two unpublished approaches to the problem. One is a series of investigations by new mathematical methods of the distributions of the r, t, and F statistics, both for large and for small samples, in various cases of non-normal and sometimes non-independent observations, with special attention to the tails of these distributions, which show very small probabilities of greater values of the statistics. The other approach is through the design of experiments, which can be carried out in such ways as to improve the approximation to normality of the observations made.

V. I. MANN (Geology): *The Lituya Bay Earthquake of July 9, 1958.*

558TH MEETING, FEBRUARY 9, 1960

K. SUGIOKA (Surgery): *The Effect of Carbon Dioxide on Brain Tissue Oxygenation.* The demonstration by Kety and Schmidt that active or passive hyperventilation resulted in decreased cerebral blood flow suggested to us the intriguing possibility that this decreased blood flow might result in cerebral hypoxia even in the presence of increased arterial oxygen saturation. Hyperventilation results in decreased carbon dioxide tension in the circulation which in turn causes cerebral vaso-constriction with diminution of blood flow through the brain. Whether this decrease in blood flow is enough to cause cerebral hypoxia was a question that could be answered only by direct measurement of the brain tissue oxygen tension. This was accomplished by the use of a recently developed oxygen microelectrode which is a miniaturized version of the "Clark" polarograph.

The microelectrode was inserted into the cerebral cortex of the dog and the oxygen tension was recorded continuously on one channel of a multichannel recorder. Simultaneous recordings were made of the alveolar carbon dioxide tension as measured by an infrared carbon dioxide analyzer and the central arterial pressure as measured by a strain gauge pressure transducer. The animals were hyperventilated successively on room air and 100% oxygen for a period of five minutes; then 10% carbon dioxide was added to the inhaled mixture. It was found that hyperventilation with either room air or 100% oxygen resulted in a steady decrease in brain tissue oxygen to hypoxic levels. This drop in oxygen tension paralleled very closely the decrease in alveolar carbon dioxide. The addition of 10% carbon dioxide to the inhaled mixture resulted in a dramatic rise in brain oxygen tension whether room air or 100% oxygen was used to hyperventilate the animal. None of these changes in oxygen tension correlated with changes in arterial pressure.

We have concluded from these studies that cerebral hypoxia may be caused by hyperventilation in the dog and that if this is true in the human, perhaps the use of hyperventilation in anesthesia and resuscitation may be harmful.

C. N. REILLEY (Chemistry): *Chemical Applications of Tristimulus Colorimetry.* The principles of tristimulus colorimetry have been extended in mathematical form to several problems of analytical interest. Screening of chemical indicators may be calculated *a priori* and the precision of end point detection is enhanced by a factor of four thereby. Colorimetric analysis of multicomponent mixtures is another possible application. While the analysis of a single component is trivial, the use of complementary tristimulus colorimetry in the case of two and three components with overlapping spectra is feasible and the technique has the advantage that more than the minimum number of wavelengths can easily be employed and better precision thereby achieved. While the tristimulus approach is limited to a maximum of three components, an analogous multi-stimulus approach can be used for a system containing more components. Other applications consist of calculation of the pK's of acids and the formula and stability constants of complex ions. To facilitate computation, an inexpensive analog computor was built which permits calculation of chromaticity coordinates (by the summation method at 10 selected wavelengths) in about two to three minutes.

559TH MEETING, MARCH 8, 1960

C. H. BLAKE (formerly of Dept. of Biology, Mass. Inst. of Technology) *Did* Archaeopteryx *Fly or Glide?* It is generally agreed that *Archaeopteryx lithographica* (Jurassic of Solnhofen, Bavaria) was arboreal and could perch without assistance from the clawed digits of the wings.

From the presently available information we may estimate a weight of 0.29 lbs., span of 1.56 ft., wing loading 0.58 lbs./sq. ft., and aspect ratio of 4.9. These closely resemble the figures for many modern birds. I agree with de Beer that the small, keeless sternum indicates wing muscles too small for flapping flight. If the bird did glide the remaining question is whether it could land without damage. My estimate of the minimum rate of sink is 2.6 ft./sec. with a velocity along the path of 21 ft./sec.

If it aimed correctly at the landing spot at minimum rate of sink then uniform deceleration over 0.04 sec. would not produce an upward force on the legs greater than the bird's own weight. If it aimed too high the deceleration time would have to be increased. By aiming below the landing spot and coasting up to it ("flaring"), granted adequate skill, landing would be possible with any desired impact, regardless of velocity before the "flare." The bird should have been able to glide.

M. H. HOMMERSAND (Botany): *On the Nature of the Light Requirement for Spore Germination in a Unicellular Green Alga,* Chlamydomonas.

560TH MEETING, APRIL 12, 1960

J. T. SESSIONS (Medicine): *Enzymologic Aspects of Human Liver Disease.*

C. E. JENNER (Zoology): *Studies on the Role of Daylength in the Control of Growth in the Resting Larvae of Various Species of Dipterous Insects.*

561TH MEETING, MAY 10, 1960

R. PENNIALL (Biochemistry): *Two, Four-Dinitrophenol and Magnesium Activation of Adenosine Triphosphatase Activity.* Acetone treatment of mitochondria is known to solubilize part of the adenosinetriphosphatase (ATPase) activity. Aqueous extracts of mitochondrial acetone powders contain ATPase activity that responds to 2:4-dinitrophenol (DNP) and Mg^{++}, and is unsedimented by 60 minutes centrifugation at 105,000 × g. Apparent optima for stimulation by DNP and Mg^{++} are at pH 8.5 and 9.0. The ATPase is stimulated by several divalent cations although Mg^{++} and Mn^{++} are most efficient. The DNP-ATPase is quite unstable, but the activity responsive to Mg^{++} is stable in vacuo at −20°C. Data obtained thus far indicate that at least a portion of the Mg^{++}-ATPase can be characterized as a flavoprotein. In the absence of added monovalent cations, the extracts exhibit three apparent peaks of activation in the presence of varying DNP concentrations. Present interpretation is that this phenomenon is indicative of three separate DNP responsive enzymes, one for each of the three sites of oxidative phosphorylation in mitochondria. (Supported in part by a grant from the National Science Foundation, the American Heart Association, and the North Carolina Heart Association.)

P. R. BURTON (Zoology): *A Cytological Study of Gametogenesis and Fertilization in the Frog Lung Fluke, Haematoloechus medioplexus.*

Recipient of the Coker Award this year, selected by the Coker Award Committee (J. C. Morrow, Chairman; J. S. MacNerney, and J. E. Wilson), was Paul Burton, Department of Zoology, for his dissertation (see title above).

Dr. J. N. Couch, Chairman of the Editorial Board of the Elisha Mitchell Scientific Society for many years, this year asked to be relieved of this position. A committee consisting of C. W. Hooker, Chairman, C. D. Beers, W. A. Bowers, G. R. MacCarthy, and R. L. McKee, appointed by President Irvin to nominate a successor to Dr. Couch, named Walter H. Wheeler of the Department of Geology and Geography, who was unanimously elected. Dr. Hooker then read a statement of thanks in behalf of the Elisha Mitchell Scientific Society to Dr. Couch for his long and devoted years of service as Editor of the *Journal of the Elisha Mitchell Scientific Society.* Dr. Hooker's Committee was asked to continue study of the financial status of the *Journal* with a view to obtaining adequate and stable support for it.

The Treasurer's report was read and the Auditing Committee (C. N. Reilley, Chairman; and A. E. Radford) reported the books and accounts in order.

The Nominating Committee (E. D. Palmatier, Chairman; G. P. Manire, W. H. Wheeler) presented the following nominees for 1960–61:

President: Alfred T. Brauer

Vice President: William J. Koch.

The slate was unanimously elected. Dr. M. R. Carriker will continue to serve as secretary-treasurer for a second year.

MELBOURNE R. CARRIKER, *Sec.-Treas.*

ABSTRACTS OF PAPERS PRESENTED AT THE SPRING MEETING OF THE SOCIETY OF NORTH CAROLINA BACTERIOLOGISTS AT CHAPEL HILL, N. C. ON APRIL 2, 1960

1. An Improved Acid-Fast Stain for *Nocardia* in Clinical Materials. *John Carr, Veterans Administration Hospital, Durham.*

2. Cross Reactions between Human, Battey and Avian Tuberculins. *David T. Smith, Duke University School of Medicine, Durham.*

3. Isolation of an Adenovirus Antigenically Related to Type 9 and 15 from Children with Respiratory Disease. *Henry G. Cramblett, Bowman Gray School of Medicine, Winston-Salem.* In March, 1958, illnesses occurred in five patients on the Pediatric Ward at University Hospitals of the State University of Iowa. The manifestations of the illnesses included fever, rhinorrhea, erythema of the posterior wall of the pharynx, cough, rhonchi, vomiting, and diarrhea.

A virus was isolated from all the patients who were ill in HeLa cell cultures. The isolates from all five patients were consistently neutralized by type 15 adenovirus antiserum. However, in hemagglutinating characteristics and in hemagglutination-inhibition tests, the isolates were all identified as type 9.

Hyperimmune serum prepared in rabbits against a prototype strain of this virus (Vance virus) consistently neutralized both the homologous virus and type 15 adenovirus, but not adenovirus type 9.

Cross antigenic relationships among certain adenoviruses have been noted by other investigators. The present study exemplifies the difficulty in categorizing an adenovirus when such cross antigenic relationships are encountered in naturally occurring strains of adenoviruses. The virus that we have described would be designated as type 15 by neutralizing antibody studies. However, by hemagglutination-inhibition tests, the Vance virus is more closely related to type 9.

4. The Demonstration by Immunoelectrophoresis of the Absorption of Unaltered Protein Through the Piglet's Intestine. *D. O. Morgan, J. G. Lecce and G. Matrone, School of Agriculture, North Carolina State College, Raleigh.* The electrophoretic serum protein pattern of the neonatal pig varies greatly from the pattern of the mature pig. The piglet is born lacking in gamma globulin, low in beta globulin, high in alpha globulin, and low in albumin. The electrophoretic serum protein pattern of the nursing piglet shows rapid changes toward the pattern of a mature pig. These changes are much faster and of larger quantity than would be expected from *de novo* synthesis. Comparison of the electrophoretic protein pattern of neonatal serum, sow colostrum, and nursed piglet serum supports the theory of a mechanism whereby direct absorption of unaltered proteins from the piglet's gut takes place. In an effort to determine the specificity of this absorption mechanism and to further demonstrate its existence, neonatal piglets were fed a diet of whole egg. Using serum from 24-hour piglets on egg diet as an antigen and anti-egg white serum, positive ring precipitin tests were obtained. To determine which fractions of the piglet's serum were of egg origin, immunoelectrophoresis was employed. Immunoelectrophoresis gives the resolving power of electrophoresis coupled with the specificity and sensitivity of serology. Using immunoelectrophoresis, it is indicated that all electrophoretic fractions of egg white are absorbed directly from the gut of the piglet serologically unaltered. Thus, one may surmise that a mechanism is present for absorption of unaltered proteins, and since avial and bovine proteins are absorbed unaltered, that this mechanism in nonspecific in relation to phylogenetic protein source.

5. Antibodies in Saliva. *H. Stoffer and John H. Schwab, University of North Carolina School of Medicine, Chapel Hill.* Previous studies on antibodies in saliva have failed to demonstrate the specific antibody properties of this factor.

In this study the saliva and serum of rats and rabbits immunized with Bovine Plasma Albumin (BPA) were subjected to various serological studies including the ring precipitin technique, the hemagglutination technique of Boyden, and the passive cutaneous anaphylaxis technique. The studies were also extended to demonstrate some of the physical-chemical characteristics of the substance by use of rivanol and ammonium

sulfate fractionation, and moving boundary electrophoresis.

The data demonstrated that an antibody-like substance can be detected in the saliva of rats, by use of hemagglutination and passive cutaneous anaphylaxis, after immunization with BPA. The route of antigen administration influences the production of this immune substance. It was further shown that this factor can be absorbed from the saliva with the specific immunizing agent (BPA). An anamnestic response was induced in the rat saliva after a booster injection of antigen. The solubility studies with ammonium sulfate and rivanol indicated that the greater percentage of salivary antibody activity lies in the globulin fraction. This observation was confirmed by electrophoretic analysis. From these results it was concluded that a true antibody can be detected in the saliva of rats immunized with a pure protein antigen. A comparable factor meeting the definition of an antibody was not detected in rabbit saliva.

6. The Effect of Dietary Protein on the Resistance of Chicks to Salmonella Gallinarum Infection. *Charles. H. Hill, School of Agriculture, North Carolina State College, Raleigh.*

7. Role of Basic Proteins in Natural Resistance to Infection. I. Cytochemical Studies on the Effect of Basic Protein on Bacteria. *John K. Spitznagel, University of North Carolina School of Medicine, Chapel Hill.* Staining with Fast Green FCF at pH 8 is selective for basic proteins such as histones (Spitznagel, J. K., Proc. Fed. Soc. Exp. Biol. **19:** 197, 1960). Extranuclear material in experimental guinea pig tubercles is stained by this stain without prior acid extraction. Some bacilli in the lesions also appear to take the stain. This suggests that an interaction between host histones and the parasite is being demonstrated since mycobacteria do not appear to synthesize histone-like proteins but are poisoned by them. (Spitznagel, J. K., Elisha Mitchell Soc. **75:** 80, 1959).

The relationship of fast green stainability to the interaction of bacteria and basic proteins was studied *in vitro* using several organisms. *Escherichia coli* and *Bacillus anthracis* proved good models and were stainable after exposure to histone at pH 5.6–6.8. Without histone they were totally resistant to the dye. Independent cytochemical techniques based on the fluorescence and high arginine content of histones confirmed that histone actually combined with the bacterial cells. Trypsinization abolished the fast green staining of histone treated cells. Deamination of bacteria did not abolish reactivity with histone as demonstrated by Fast Green staining. Bacteria treated with albumin globulin, ribonuclease, lysozyme, penicillin, and polymyxin were not stainable with Fast Green.

The percentage of cells stainable by fast green was proportional to the percentage of cells killed by histones and to the suppression of bacterial respiration by histones. Mucin inhibition of bacterial interaction with histone was reflected by proportional changes in fast green staining and bacterial killing.

It appears that basic proteins interact with bacteria in infected tissues as well as *in vitro*. The interaction is demonstrable by Fast Green which should prove useful in studying the role of basic proteins in nonspecific resistance.

8. Role of Basic Proteins in Natural Resistance to Infection. II. Participation of Basic Proteins in Host Parasite Interaction in Actinomycosis. *W. J. Cromartie, J. K. Spitznagel and J. J. Crawford, University of North Carolina School of Medicine, Chapel Hill.* Bacteria that have taken up histone have been shown to stain with Fast Green at pH 8. (Spitznagel, J. K., Fed. Proc. **19:** 197, 1960). Staining of bacteria and killing were found to be roughly parallel. Histological sections of lung tissues from two cases of actinomycosis in human beings were stained by a variety of methods including Fast Green at pH 8 and the Sakaguchi oxine test for arginine. The results suggest that there is an accumulation of a basic protein with a high arginine content at the periphery of "sulfur granules." This protein appears to be in and around the radially arranged clubs which, when stained by other methods, take the eosin stain. *In vitro* studies indicate that the basic protein histone will inhibit the growth of a strain of *Actinomyces israelii* and inhibition of growth was found to be associated with staining by Fast Green at pH 8. These findings support the idea that proteins with high isoelectric points play an important role in the host-parasite interaction in actinomycosis and indicate that it will be possible to make a more precise evaluation of the role of these proteins in natural resistance to this disease.

MARINE SPONGES OF NORTH CAROLINA[1]

By Harry W. Wells, Mary Jane Wells, and I. E. Gray

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Introduction

Although the sponge fauna of North Carolina has in the past received the attention of such authorities as M. W. de Laubenfels, W. C. George, and H. V. Wilson, recent collections from this coast have revealed many gaps in our knowledge of this phylum. In this report we have attempted to organize and correlate the collections of these and other workers, including our own, and to present information in a form that will be useful for the identification of North Carolina sponges with a minimum of reference to other works. This study is the result of many personal collections, analyses of collections of previous workers, extensive consultation of sponge literature, and comparisons with museum specimens.

Historical Review

The earliest published enumeration of marine sponges from North Carolina is that of Coues and Yarrow (1879) who, from specimens collected at Beaufort Inlet near Fort Macon, listed those species identified by Verrill. Because there are no descriptions and few identifying remarks accompanying their names, this work is of no value in sponge identification. In 1919, George and Wilson described seventeen forms in an excellent paper on sponges in the Beaufort area. Actually there were sixteen species, for one was discussed twice under different names. Although de Laubenfels (1947) attempted to bring the taxonomy of these sixteen species up to date, the lack of representatives of certain species resulted in much synonymizing. The species described by George and Wilson are here listed with corrections.

1. *Spirastrella andrewsii*, new = *Spheciospongia vesparia* (Lamarck)
2. *Cliona celata* Grant
3. *Poterion atlantica*, new = *Spheciospongia vesparia* (Lamarck)
4. *Suberites undulatus*, new = *Cliona viridis* (Schmidt)
5. *Tetilla laminaris*, new = *Craniella laminaris* (George & Wilson)
6. *Reneira tubifera*, new = *Adocia tubifera* (George & Wilson)
7. *Stylotella heliophila* Wilson = *Hymeniacidon heliophila* (Parker)
8. *Esperiopsis obliqua*, new = *Tenaciella obliqua* (George & Wilson)
9. *Lissodendoryx carolinensis* Wilson = *Lissodendoryx isodictyalis* (Carter)
10. *Phloeodictyon nodosum*, new = *Rhizochalina oleracea* Schmidt
11. *Phoriospongia osburnensis*, new = *Xytopsues griseus* (Schmidt)
12. *Microciona prolifera* Verrill = *Microciona prolifera* (Ellis & Solander)
13. *Axinella acanthifera*, new = *Endectyon tenax* (Schmidt)
14. *Acanthella corrugata*, new = *Higginsia strigilata* (Lamarck)
15. *Aplysilla longispina*, new = *Aplysilla sulfurea* Schulze
16. *Pteraplysilla latens*, new = *Pteraplysilla minchini* Topsent
17. *Hircinia ectofibrosa*, new = *Ircinia fasciculata* (Pallas)

In his study of boring sponges along the Atlantic coast, Old (1941) collected in the Beaufort area and added four species of the genus *Cliona* to the sponge fauna of the state: *Cliona lobata* Hancock, *C. robusta* Old, *C. spirilla* Old, and *C. vastifica* Hancock.

De Laubenfels (1947) added several more species for the Beaufort area:

1. *Haliclona permollis* (Bowerbank)
2. *Haliclona excelsa* (Schmidt) = *Haliclona areolata* (Wilson)
3. *Craniella crania* Müller
4. *Calyx poa*, new = *Halichondria bowerbanki* Burton

Pearse and Williams (1951) published a list of sponges from reefs located off New River Inlet, North Carolina. These were identified by de Laubenfels. We have been able to examine most sponges of this collection. The published list, with corrections, follows:

1. *Spongia graminea* Hyatt
2. *Phyllospongia lanuga* de Laubenfels
3. *Ircinia fasciculata* (Pallas)

[1] This research was supported by grant G-5838 from the National Science Foundation.

4. *Ircinia campana* (Lamarck)
5. *Dysidea* sp. = *Dysidea fragilis* (Montagu)
6. *Haliclona ?coerulescens* (Topsent) = *Haliclona areolata* (Wilson)
7. *Haliclona oculata* (Linné) = *Tenaciella obliqua* (George & Wilson)
8. *Microciona prolifera* (Ellis & Solander) = *Microciona juniperina* (Lamarck)
9. *Thalyseurypon carteri* (Topsent) = *Homaxinella waltonsmithi* de Laubenfels
10. *Thalyseurypon foliacea* (Topsent) = *Thalyseurypon carteri* (Topsent)
11. *Myrmekioderma* n. sp. = *Higginsia strigilata* (Lamarck)
12. *Higginsia coralloides* Higgin = *Higginsia strigilata* (Lamarck)
13. *Phakellia folium* Schmidt
14. *Homaxinella* sp. = *Teichaxinella lata*, new
15. *Pseudaxinella* sp. = *Pseudaxinella rosacea* (Verrill)
16. *Hymeniacidon heliophila* (Parker) = *Oxeostilon burtoni* de Laubenfels
17. *Ciocalapata* n. sp. = *Ciocalapata gibbsi*, new
18. *Pseudosuberites melanos* de Laubenfels
19. *Cliona caribboea* Carter
20. *Dorypleres* n. sp. = *Dorypleres carolinensis*, new
21. *Myriastra fibrosa* (Schmidt)
22. *Geodia gibberosa* Lamarck
23. *Craniella crania* Mueller = *Trachgellius cinachyra* de Laubenfels
24. *Cionchyra cavernosa* (Lamarck) = *Cinachyra cavernosa* (Lamarck)
25. *Lucetta floridana* (Haeckel) = *Leucetta floridana* (Haeckel)

In addition, *Cliona truitti* Old has been reported in North Carolina waters by Wells (1959).

Of these works, only those of George and Wilson and of Old are particularly useful for identification purposes. Most of the sponges covered in the former paper are also figured in Fieldbook of Seashore Life by R. W. Miner (1950), but their nomenclature required considerable revision, as de Laubenfels pointed out in 1947. Because both northern and southern sponges occur in North Carolina, it has been necessary to use numerous works for identification. In particular, for northern species, the papers on sponges of the New England region by Hartman (1958) and by de Leubenfels (1949a) have been very useful; and for southern species, the publications by de Laubenfels on the sponges of the Dry Tortugas (1936b), of Bermuda (1950a), and of the Gulf Coast of Florida (1953) have been particularly helpful.

MATERIAL

Much of the material treated in previous studies of North Carolina sponges has been made available to us. Through the cooperation of Dr. M. R. Carriker and the Department of Zoology, University of North Carolina, we have examined a collection of sponges made by H. V. Wilson from the Beaufort area. Through the courtesy of Mr. D. E. Giles and the Department of Zoology, Oregon State College, we have examined the collection of sponges from the Beaufort area made in 1946 by de Laubenfels; and through the cooperation of Dr. C. G. Bookhout of the Department of Zoology, Duke University, we have examined the sponges from New River fishing banks collected by Pearse and Williams and identified by de Laubenfels. In addition to these, extensive collections have been made by the authors in the Beaufort and Cape Hatteras regions. A large number of the species collected represent new records for North Carolina.

Collections at the United States National museum have been consulted with the valuable assistance of Dr. C. E. Cuttress of the Division of Marine Invertebrates. Inquiries made to the American Museum of Natural History (New York), Academy of Natural Science (Philadelphia), Peabody Museum of Natural History (New Haven), Charleston Museum (Charleston), and the British Museum (London) for sponge collections from the North Carolina area, produced small lots duplicating other materials already available.

For species described here as new, type material has been deposited in the United States National Museum, Washington, D. C. Whenever possible, representative specimens of all species have been deposited in the U. S. National Museum, and are indicated by "U.S.N.M." and a collection number.

METHODS

Sponges are, of course, best studied in a living condition. Preservation in alcohol is next best, with drying preferred over preservation in formalin for long periods of time. Before preservation, notes on color and consistency are usually important, for these characters will often change.

In the best procedure, a fresh sponge should be placed into 95% alcohol for several hours to allow time for hardening of the tissues and the precipitation of certain insoluble complexes. Permanent storage can then be made in clean 70% alcohol.

The form of a sponge colony is often of primary importance in recognizing species in the field, and indeed, is often of great value to a taxonomist for its identification. Although growth form may be a subject to a certain range of ecological variation, it generally is stable enough to serve as an important taxonomic character. Some workers have placed complete reliance on the size and shape of spicules as criteria for the identification of sponges; the variability of spicule shapes and sizes and the occasional absence of spicules of a particular type have consequently resulted in a number of misidentifications. The British spongologist, Maurice Burton, has written (1932): "As a result of my own experience, I am coming to the conclusion that, for the identification of species, the external form is a more reliable feature than the skeleton; that it is more constant than the shape of the spicules, and far less variable, under normal conditions, than is usually assumed." Actually, no one character is sufficient for sponge identification, but rather a combination of characters, including spiculation, growth form, color, and life history. However, recognizing the validity of Burton's remarks, we have included illustrations of the growth form for as many North Carolina species as possible, omitting only those which are thin, featureless encrustations or which are extremely plastic species.

One character that may vary with ecological conditions is the number of oscules on a sponge and the relative development of chimneys around them. Apparently this is related to their function as excurrent channels that carry excretory materials and water from which nutritive particles have been captured. In quiet waters where circulation is minimal, sponges show a maximum number of oscules per volume of sponge tissue and they may be elevated on chimneys well above the incurrent pores distributed over the surface of the sponge body. This arrangement confers obvious advantages to the sponge. On the other hand, sponges growing in surf or strong currents may have fewer and less conspicuous oscules. Common species that illustrate this type of variation include *Lissodendoryx isodictyalis* and *Haliclona loosanoffi*.

Another type of variation in form in different habitats is shown by the common northern sponge *Microciona prolifera*. Generally, this species forms only thin encrustations in the more exposed situations in high salinities. Just outside Hatteras Harbor it may form lamellate processes, and on the pilings within that harbor it produces beautiful head-sized masses of erect, finger-like lobes.

Usually, spicules are essential for final identification. Together with lesser or greater amounts of spongin, they comprise the skeleton of a sponge. Because their shapes and sizes are diagnostic features, they must be freed from surrounding tissue for microscopic examination. This is most easily accomplished by placing a tiny piece of sponge in a drop of sodium hypoclorite (Clorox) on a glass slide. When bubbling has ceased, the spicules can be viewed microscopically, but care must be taken to avoid contact between the basic fluid and the microscope. Calcareous spicules can be recognized by the effect of acid upon them: they will dissolve, while siliceous spicules will not. Spicule types are shown in Figure 1. The types found and their dimensions are included in descriptions and are illustrated for many species.

Several factors that affect the size of spicules should be mentioned. In many sponge preparations, extremely slender spicules can be found that represent early stages in the development of normal spicules; there is usually a series of intermediate stages relating these slender forms to thicker mature spicules. Ranges of spicule sizes given in descriptions do not include the extremely thin forms that clearly represent growth stages.

In addition, there is a clear-cut correlation of spicule size with water temperature, with the larger size in colder northern waters. For example, Hartman (1958) found for *Haliclona oculata* that the average measurements of oxeas from colonies north of Cape Cod were 133 μ long and 8.8 μ wide, while those from colonies south of Cape Cod were 114 μ long and 7.8 μ wide. Hentschel (1929) showed such a correlation of spicule length with water temperature within several genera. These observations indicate that spicular sizes may vary within the range of certain species, with an increase in size more likely in colder waters. This is particularly important in interpreting the differences in spicule sizes in specimens from North Carolina compared with spicules in specimens from the West Indies for

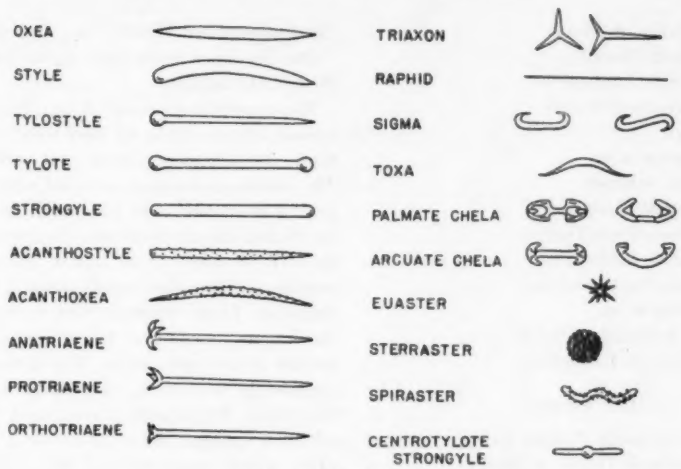


FIG. 1. Spicule types.

example; quite often, North Carolina representatives of northern species may possess slightly smaller spicules than is typical of that species at a locality farther north. Such ecophenotypic variation apparently occurs in several species of *Haliclona*. This type of variation must be kept in mind when one compares North Carolina specimens with descriptions of sponges from other localities.

Often the placement of spicules in the sponge is of importance also. Some sponges contain distinct fibers composed of a horny substance (spongin) that completely surrounds the individual spicules, while others possess structures that are best described as spicule tracts, in which bundles of spicules can be seen with very little spongin material present. The organization of spicules in these and similar patterns that are characteristic of certain groups can be readily observed by mounting small sections in Hoyer's mounting medium. Of course, permanent slides of spicules and sections can be made in balsam.

I. Systematic Treatment

by Harry W. Wells and Mary Jane Wells

Species, listed in the order of treatment are:

DEMOSPONGIA

Keratosa

Spongia graminea Hyatt
Phyllospongia lamuga de Laubenfels
Ircinia campana (Lamarck)
I. fasciculata (Pallas)
Dysidea fragilis (Montagu)
Aplysilla sulfurea Schulze
Pleraplysilla minchini Topsent

Haplosclerina

Haliclona areolata (Wilson)
H. canaliculata Hartman
H. loosanoffi Hartman
H. palmata (Ellis & Solander)
H. permollis (Bowerbank)
H. viridis (Duch. & Mich.)
Xestospongia halichondrioides (Wilson)
Callyspongia vaginalis (Lamarck)

Poecilosclerina

Adocia tubifera (George & Wilson)
Rhizochalina oleracea Schmidt
Lissodendoryx isodictyalis (Carter)
Mycale cecilia de Laubenfels
Xytopsues griseus (Schmidt)
Microciona prolifera (Ellis & Solander)
M. juniperina (Lamarck)
Thalyseurypon carteri (Topsent)
Eurypon clavata (Bowerbank)
Dictyociona adioristica de Laubenfels
Endectyon tenax (Schmidt)
Hemectyon pearsei n. sp.
Tenaciella obliqua (George & Wilson)

Halichondrina

Axinella bookhouti n. sp.
A. polycapella de Laubenfels

A. reticulata Ridley & Dendy
Homaxinella rudis (Verrill)
H. waltonsmithi de Laubenfels
Pseudaxinella rosacea (Verrill)
P. wilsoni n. sp.
Teichaxinella grayi n. sp.
Phakellia folium Schmidt
Higginsia strigilata (Lamarck)
Halichondria bowerbanki Burton
H. melanadocia de Laubenfels
Ciocalypta penicillus Bowerbank
Ciocalapata gibbsi n. sp.
Hymeniacidon heliophila (Parker)
Oxeostilon burtoni de Laubenfels

Hadromerina

Spheciospongia vesparia (Lamarck)
Spirastrella coccinea (Duch. & Mich.)
Anthosigmella varians (Duch. & Mich.)
Suberites ficus (Johnston)
Pseudosuberites sulphureus (Bowerbank)
Prosuberites microsclerus de Laubenfels
Terpios fugax Duch. & Mich.
Polymastia robusta Bowerbank
Cliona caribboea Carter
C. celata Grant
C. lobata Hancock
C. robusta Old
C. spirilla Old
C. truitti Old
C. vastifica Hancock
C. viridis (Schmidt)

Epipolasida

Dorypleres carolinensis n. sp.
Trachygellius cinachyra de Laubenfels

Choristida

Myriastra fibrosa (Schmidt)
Geodia gibberosa Lamarck
Cinachyra cavernosa (Lamarck)
Craniella crania (Müller)
C. laminaris (George & Wilson)

CALCISPONGIA

Scypha barbadensis (Schuffner)
Leucetta floridana (Haeckel)
Leucosolenia canariensis (Miklucho-Maclay)

●

DEMOSPONGIA

ORDER KERATOSA

Skeleton of spongin fibers. Sand and foreign spicules (often broken) may be embedded in the fibers of some species.

Spongia graminea Hyatt Key grass sponge

One specimen from New River fishing banks (U.S.N.M. 23652).

This sponge is represented by a piece of curved lamina 54 mm high, 35 mm wide, and 13 mm thick. It is brown colored and stiffly spongy. The surface is conulose, covered with stout, projecting fiber ends over which fine, narrow fibers are draped like a spider web. Conules often occur in vertical ridges 2 to 3 mm apart. Between conules are abundant small pores, 1 to 2 mm in diameter. Large channels run vertically within the lamina, opening at the upper edge through oscules 3 to 6 mm across. The dermis is almost completely destroyed; apparently it was about 25 μ thick. The interior is composed of a network of small spongin fibers 25 to 40 μ in diameter (Fig. 3), and many vertical fibers 30 to 100 μ in diameter, which may contain foreign debris in their core.

This species typically forms columnar colonies 12 to 25 cm in diameter in which the base is smaller and the upper surface is truncate (de Laubenfels & Storr 1958). Oscules as large as 1 cm in diameter occur on the upper surface, and may be surrounded by thin chimneys 1 cm high in which the coarse vertical fibers are conspicuous.

This species was first described by Hyatt (1877, p.516). It is typically a West Indian sponge.

Phyllospongia lanuga de Laubenfels

This species has been reported from New River fishing banks (Pearse & Williams 1951), but no specimens have been available for examination.

It was described by de Laubenfels (1936b) as an irregularly lobate mass, marked by dome-shaped protrusions 1 to 3 cm in diameter. It is gray in color, and stiffly elastic. The surface is hispid with protruding fibers. A tough cortex, 600 to 1200 μ thick, is packed with sand and foreign spicule fragments, all about 20 μ in greatest dimension. There are extensive subdermal spaces. Oscules are conspicuous, 2 to 5 mm in diameter. Internal fibers, 30 to 45 μ in diameter, branch and anastomose to form a network with meshes 125 to 700 μ across. These fibers rarely have foreign material embedded in them, although foreign material occurs in the flesh.

This species was originally described as *Coscinoderma lanuga* by de Laubenfels (1936b, p.10), and was transferred to the genus *Phyllospongia* by de Laubenfels (1948, p.52).

Ircinia campana (Lamarck) de Laubenfels

One specimen from New River fishing banks.

This species is consistently cup-shaped and

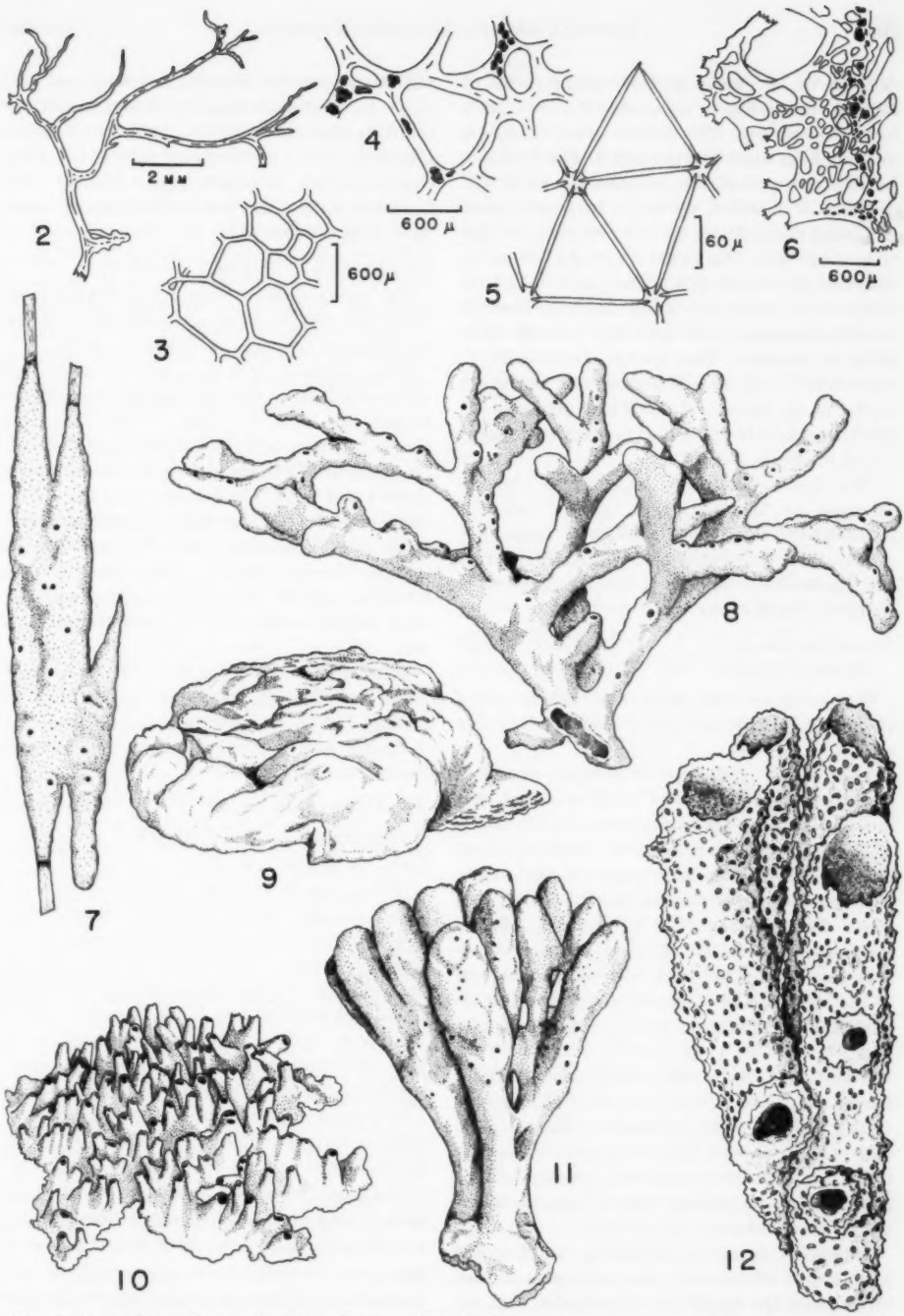


FIG. 2. *Aplysilla sulfurea*, dendritic fiber. FIG. 3. *Spongia graminea*, superficial fibrous network. FIG. 4. *Dysidea fragilis*, fibers containing foreign material. FIG. 5. *Haliclona permollis*, unispicular reticulation. FIG. 6. *Ircinia fasciculata*, fascicular fiber containing some foreign material. FIG. 7. *Haliclona palmata*. FIG. 8. *Haliclona viridis*. FIG. 9. *Xestospongia halichondrioides*. FIG. 10. *Haliclona loosanoffi*. FIG. 11. *Haliclona areolata*. FIG. 12. *Callyspongia vaginalis*.

marked by a coarse surface arrangement of conules. It exhibits a brownish-red color and a tough consistency. The surface bears conules 2 to 4 mm high and 3 to 8 mm apart. The dermis is perforated by numerous conspicuous pores 125 to 200 μ in diameter. Oscules 1 to 7 mm across are found primarily on the concave surface. The tissue bears many filaments 2 to 4 μ in diameter with ball-like terminations. The primary skeletal elements are unusually broad sheets of fiber 50 to 130 μ in diameter; the secondary ones are 15 to 35 μ in diameter. This sponge shares with *I. fasciculata* a fetid odor, the abundance of foreign matter in the fibers, and the occurrence of peculiar filaments in the flesh—all characteristics typical of the genus *Ircinia*.

This species was first described as *Spongia campana* by Lamarck (1813, p.385), and was transferred to the genus *Ircinia* by de Laubenfels (1948, p.71). It has also been treated as *Hircinia* (de Laubenfels, 1936b). It is primarily West Indian in distribution.

Ircinia fasciculata (Pallas) Schmidt — Stinker sponge

One specimen from New River fishing banks (U.S.N.M. 23563), and several from the Beaufort area.

This species produces finger-like processes marked by vertical rows of small conules 1 to 2 mm high that are 1 to 4 mm apart. By the fusion of these upright lobes, an erect, massive colony may be formed. These later stages are represented by single specimens from Beaufort and from New River fishing banks (U.S.N.M. 23653). Beach specimens may be an off-white or pale yellow color, although a brownish color is more typical of the species. The consistency is tough and resistant. Fresh specimens have a distinctive garlic odor.

The dermis, 70 to 100 μ thick, often contains foreign debris as well as typical *Ircinia* filaments —long filaments with spherical swellings at each end. Large oscules, darker than surrounding tissue, are obvious on massive specimens but are inconspicuous on younger stages. Pores are abundantly scattered over the surface.

Internally, main longitudinal fibers 150 to 800 μ thick give off branches that radiate to the surface and lift the dermis into the conules. Actually many of these fibers are sheets or bundles of fiber (Fig. 6). They often contain foreign debris.

This species was first described as *Spongia fasciculata* by Pallas (1766, p.381). De Laubenfels (1936b) treated it as *Hircinia variabilis*, and early stages have been described by George and Wilson (1919, p.166) and Miner (1950) as *Hircinia ectofibrosa*. De Laubenfels explained the name changes in his Bermuda paper (1950a). This species is distributed from North Carolina southward as far as Panama.

Dysidea fragilis (Montagu) Johnston

One specimen from New River fishing banks (U.S.N.M. 23654). It has also been reported from the Beaufort area (Coues & Yarrow 1879).

This specimen is a gray-colored mass from which arise hollow processes, about 1 cm high and in diameter, that bear large oscules 3 to 7 mm across. It is 5 cm high. The entire mass is soft and compressible. Upon its surface are minute conules about 1 mm apart. The dermis is 20 μ thick, overlying extensive cavities and a fibrous endosome. In the interior, primary fibers often contain much sand and broken spicules and are usually 100 to 200 μ in diameter (Fig. 4). Smaller fibers (under 50 μ) may be relatively free of such foreign material.

This species was first described as *Spongia fragilis* by Montagu (1818, p.114). De Laubenfels (1936b, 1950a) has redescribed specimens from Tortugas and Bermuda. It is a typical West Indian form.

Aplysilla sulfurea Schulze — Sulfur sponge

Represented by U.S.N.M. 23641, collected at Beaufort by Dr. I. E. Gray.

This sponge is an irregular, amorphous mass 9 by 6.5 cm and up to 3 cm thick. It was bright yellow colored and very soft in consistency. The surface is composed of a smooth dermis stretched over the tips of projecting skeletal fibers to form prominent conules 3 to 5 mm high and 4 to 10 mm apart. Because of the contraction of the dermis with preservation, the fiber ends project several millimeters beyond the conules. Oscules 1 to 2 mm in diameter are present on the upper surface, but no pores are visible. The skeletal fibers branch but do not coalesce, exhibiting a characteristic dendritic form (Fig. 2). They arise from a basal plate of spongin as a fiber about 200 μ in diameter, narrow to about 30 μ at the conular tips. They are dark brown, translucent, obviously laminated, and free of foreign matter. In death, the sulfur-yellow color of this sponge turns to an indigo or purple, which is especially evident in preservatives.

This species is more often seen as small, young

specimens that encrust shell or rock. Their slippery, shiny yellow surface is distinctive.

This species was first described by Schulze (1878, p.404). De Laubenfels (1947, 1950a) suggested that it is conspecific with *Aplysilla longispina* George and Wilson (1919, p. 163), described from the Beaufort area. The type specimen of *A. longispina* (U.S.N.M. 23608) has been examined and found to agree with characteristics of this species from other areas. *Aplysilla longispina* George & Wilson is hereby placed into the synonymy of *A. sulfurea*. This species is also treated by Miner (1950) as *A. longispina*. Typically, it is found in the West Indian region.

Pleraplysilla minchini Topsent

Two specimens (U.S.N.M. 23609) from Beaufort Harbor, and one from off Ocracoke Island.

This species is represented by thin colorless encrustations 1 to 2 mm thick. The largest specimen is 6 cm in diameter. The surface is marked by many acute conules 0.5 to 1.5 mm high and 0.5 to 3 mm apart. The dermis, about 20 μ thick, is extended over projecting fibers to form these conules. Oscules 200 to 350 μ in diameter lead into subdermal spaces. The interior region is dense. Clusters of flagellated chambers 50 to 120 μ in diameter can be found.

Extending to the surface conules are spongin fibers that are principally non-branching and simple, measuring up to 250 μ in diameter near the base, and tapering to 40 to 60 μ near the conular tip. In thick portions of the sponge, however, side branches 40 to 60 μ in diameter may develop; these never fuse with other fibers. At the base of the fiber is an expanded spongin plate that was attached to the substrate. The basal portion of primary fibers contains dark foreign material, while other parts show a distinct lamination of the yellow spongin.

This species was first described by Topsent (1905, p.clxxxv) from a depth of 30 meters off the coast of France. George and Wilson (1919, p.165) have described *P. latens* from Beaufort Harbor, with larger flagellated chambers and smaller conules, fibers, and other features being the chief differences. We have examined the syntypes of *P. latens* (U.S.N.M. 23609); they are thin, less-developed representatives of *P. minchini*. Although George and Wilson emphasized the simple nature of the spongin fibers, they did note occasional side branches, typical of *P. minchini*. In addition, we have observed small flagellated chambers (about 50 μ in diameter) in the Beaufort material, well within the size range described for *P. minchini* from France. The Ocracoke specimen is better developed and contains conules, fibers, and other structures that are as large as those decribed for *P. minchini*. With the discovery of small flagellated chambers in Beaufort material, and the collection of a specimen sharing features with both forms, *P. latens* must fall to the synonymy of *Pleraplysilla minchini*. The Ocracoke specimen was collected from a depth of 10 meters, while the Beaufort specimens were collected in the intertidal zone. Apparently age or ecological conditions were responsible for the differences in the Beaufort specimens.

This is only the third report of this species known to science.

ORDER HAPLOSCLERINA

Skeleton of silicious spicules and spongin, the spicules being primarily of one kind—oxeas, i.e. with points at both ends. Modifications of this spicule form may produce styles or strongyles as a small percentage of the total spiculation.

Haliclona areolata (Wilson), new combination

One specimen from New River fishing banks, and one (U.S.N.M. 23649) from off Beaufort Inlet.

The first specimen is a crust from which rise cylindrical or club-shaped lobes 5 to 18 mm in diameter. The entire colony is 47 mm high and 32 mm long. The second is an erect branching colony 10 cm high that rises from an expanded attachment to a shell (Fig. 11). From a basal stalk 2 cm across rise several primary branches which quickly divide to form short swollen digitate processes. These processes are 0.8 to 1.7 mm in diameter and are often attached to adjacent processes, either because of incomplete separation or anastomoses. The consistency is stiffly spongy and the coloration is a rich brown in alcohol and light brown in the dried specimen.

The surface appears punctate from evenly distributed pores 80 to 120 μ in diameter. Irregularly scattered over the surface are oscules 0.5 to 2.0 mm in diameter, some of which are closed by a thin sphincter-type membrane. Between these, the surface is formed by a layer of closely-packed fiber ends and spicule tufts. In protected areas, a thin translucent dermal membrane 20 to 25 μ thick can be found with only the tips of a few spicules projecting through it.

The skeleton is composed of a reticulation of multispicular fibers and scattered individual

spicules. The interior contains many parallel fibers 40 to 100 μ in diameter that run toward the surface. Transverse fibers, anastomoses, and individual transverse and projecting spicules are common. Near the surface, these tracts branch and narrow to about 30 μ. The spicules are oxeas 180-320 $\times$ 6-10 μ, usually somewhat curved.

This species was first described from Puerto Rico as *Pachychalina areolata* by Wilson (1902, p.392), and has also been described from Bermuda as *Pachychalina millepora* by Verrill (1907, p.336). De Laubenfels (1947) identified the Beaufort specimen as *Haliclona excelsa*, a form described by Schmidt (1870, p.37) from northwestern Europe. Unless a careful comparison of specimens from both regions shows these to be the same, it seems best to consider these forms distinct in view of the vagueness of Schmidt's description and in view of the southern distribution of *H. areolata*. De Laubenfels also identified the specimen from New River fishing banks as *Haliclona ?coerulescens* (Pearse & Williams 1951).

Haliclona canaliculata Hartman

Several specimens from the Cape Hatteras beach, and one (U.S.N.M. 23629) from off Portsmouth Island, south of Cape Hatteras.

This species forms encrustations 2 to 4 mm thick upon hydroid stems, mollusc shells, and similar substrates. In life, these colonies are a brownish gray in color, and softly spongy. Most of these North Carolina specimens are somewhat atypical, apparently due to a commensal association with polychaetous annelids of the family Syllidae. Numbers of these small annelids inhabit the sponge tissue, so that the exact nature of channels in the sponge tissue is uncertain.

At the surface a thin dermis is raised over an extensive subdermal space by projecting spicular tracts. Although this dermis is free of spicules, it does contain accumulations of mud and fecal matter from the annelids. These accumulations obscure much of the anatomy of this sponge. Cylindrical tubes, 400 μ in diameter and formed of a spongin membrane, perforate the surface. These probably were formed around passageways frequented by the annelids, for the oscules of this species are typically 0.8 to 1.4 mm in diameter. Pores are microscopic. Peripheral regions contain much flesh distributed over the skeleton.

The skeleton consists of branching multispicular tracts with scattered free and connecting spicules. The spicules are cemented together by varying amounts of spongin. The spicules are oxeas 105–128 $\times$ 4–6 μ that taper gradually toward their ends. Occassional malformed spicules have been observed that lack points on one or both ends.

This species has been described by Hartman (1958, p.73) from New England waters.

Haliclona loosanoffi Hartman

Many specimens (including U.S.N.M. 23631) from Hatteras Harbor, and one from Beaufort Harbor.

This species forms encrustations on submerged surfaces from which hollow, branch-like oscular chimneys may arise (Fig. 10). The figured specimen is an encrustation 73 by 64 mm and basically 3 mm thick, with tapering oscular chimneys extending to 10 mm above its surface. North Carolina specimens typically are a salmon or tan color in life. They are soft in consistency and easily torn. The surface is marked by projecting spicular tracts. A very thin dermis free of spicules occurs between these projecting tracts, but is easily lost. Oscules are 1 to 2 mm in diameter.

The skeleton consists of vertical multispicular tracts connected by horizontal individual spicules, forming a loose network. These tracts may be entirely encased in spongin or only connected by spongin. The spicules are oxeas 80–100 x 2–6 μ. Most specimens have spicules that are quite stout, nearly cylindrical to within a short distance from their points. Although Hartman (1958) described the spicules of this species as tapering gradually, all but one of those shown in his Figure 23 are this stout, abruptly-pointed type of oxea. Gemmules have been found in several North Carolina specimens.

This species was first described in 1958 by Hartman (p.62), who reported collections from Connecticut and Maryland.

This species may be distinguished from similar-appearing specimens of *Haliclona permollis* by the presence of spicular tracts and the smaller spicules.

Halicolona palmata (Ellis & Solander) Burton

One specimen from Hatteras Harbor and six others (including U.S.N.M. 23626) from the Pamlico Sound shore of Hatteras Island.

This species forms encrusting colonies on marine vegetation: several specimens were attached to blades of eelgrass (*Zostera*) (Fig. 7), and one was attached to an algal holdfast. The figured specimen is 8 cm long and contains an erect, finger-like process 3 to 4 mm in diameter. This sponge was

softly spongy in consistency and pinkish brown in color when fresh. It has faded little in preservative.

The surface is composed of an evenly developed plush of projecting fiber ends. Oscules 1 to 3 mm in diameter are scattered irregularly over the surface, and often occur within slightly elevated rims.

The skeleton is a fibrous reticulation composed of radiating fibers 12 to 22 μ in diameter, and connecting transverse fibers 7 to 12 μ in diameter in a ladder-like arrangement. The resulting delicate network is composed of rectangular meshes. These fibers are primarily composed of spongin with only a central core of spicules. The spicules are unusually small oxeas 52–60 x 1–2 μ.

This species was first described as *Spongia palmata* by Ellis and Solander (1786, p.189), and was transferred to the genus *Halicolna* by Burton (1930b, p.511). On the Atlantic coast, it has been collected in New England (de Laubenfels 1949a).

The growth form of this species may be simulated by other species of *Haliclona—canaliculata, loosanoffi,* and *permollis,* but its extremely small oxeas serve to distinguish *H. palmata.*

Haliclona permollis Volcano sponge
(Bowerbank) de Laubenfels

Many specimens from Beaufort Harbor.

This species is basically encrusting, but it may form hollow or solid cylindrical branches that are often fused at points of contact. One specimen contains several solid branches 3 to 5 mm in diameter and bears seven oscules only slightly elevated above the sponge surface. Another (U.S.N.M. 23636) is an irregularly lobed mass 35 by 22 mm that measures about 10 mm at its thickest part; from it rises a hollow branch or oscular chimney 10 mm long.

The surface is superficially smooth with prominent oscules 2 to 3 mm in diameter often occurring at the tip of a hollow branch or surrounded by a collar 1 to 3 mm high. Microscopically the surface is minutely tuberculate, with groups or individual spicules projecting from the tubercules. The skeletal unispicular reticulation is most clearly seen at the surface, a network of individual spicules with spongin only at their tips (Fig. 5). Abundant pores 50 to 100 μ across occur at the surface, in spaces between spicules. There is no easily separable dermis, nor subdermal cavity. The unispicular reticulation is regularly developed in peripheral regions, but becomes coarser internally, where larger spaces occur. The spicules in these two specimens are oxeas 110–130 x 3–5 μ, but other Beaufort representatives of this species contain oxeas 150–170 x 6–7 μ. This difference is probably associated with age or environmental differences.

This species was first described from Great Britain as *Isodictya permollis* by Bowerbank (1866, p.278). De Laubenfels (1947) found it common in Beaufort Harbor, and assumed that the sponge described as *Reneira tubifera* by George and Wilson (1919, p.145) was the same. However, Hartman (1958, p.71) has called attention to the differences and placed the latter species in the genus *Adocia.* The two species are indeed difficult to separate—displaying similar spiculation, color, and often growth form. *Adocia tubifera,* however, differs by possessing a distinct dermis, which bears oxeas smaller than those of the interior, and few pores; an extensive subdermal space; and many fibers that run longitudinally in the branches, and that are often visible in its thin oscular chimneys.

Haliclona permollis has been collected both north and south of North Carolina in the western Atlantic.

Haliclona viridis (Duchassaing & Michelotti) de Laubenfels

One specimen (U.S.N.M. 23642) collected on the beach at Nags Head, north of Cape Hatteras.

This sponge is a branching colony 15 cm high and 30 cm long in which the branches are 15 to 25 mm in diameter (Fig. 8). Where branches came into contact, they are fused. In life this species is yellowish-green, but upon death it fades to an off-white. It is softly spongy in consistency, and easily broken and surprisingly light-weight when dry.

The surface is relatively smooth and is composed of a minute reticulation. Oscules 1 to 3 mm in diameter are abundant on the upper surfaces of branches, usually at the summit of a knob or low mound. Microscopically the surface contains closely packed tufts of spicules 80 to 120 μ apart. Under the surface, this spicular reticulation becomes coarser, individual meshes being up to 160 μ across.

The internal skeleton is composed of reticulation of multispicular tracts with little spongin, and a connecting spicular network in the peripheral region. These multispicular tracts are predominantly longitudinal. Crossing them are rings of densely packed spicules wihch are obvious in

cross-sections. These rings probably represent earlier growth stages. The spicules are oxeas 130–186 x 3–12 μ, the majority being 150–170 x 10–12 μ.

This species was first described as *Amphimedon viridis* by Duchassaing and Michelotti (1864, p.81), and was transferred to the genus *Haliclona* by de Laubenfels (1936b, p.42). Verrill (1907, p.337) also described it as *Pachychalina micropora*. In the West Indies where it is common, it occurs as an encrusting, massive, or branching sponge. Its spicules have been reported as 100–150 x 3–9 μ at Tortugas and Bermuda. The greater size attained in this North Carolina specimen can be attributed to lower water temperatures on this coast.

Xestospongia halichondrioides (Wilson) de Laubenfels

One specimen from Ocracoke Island (U.S.N.M. 23622), and one from Beaufort Harbor.

This species is represented by a flattened massive sponge 4 by 3 cm from which low mounds rise, making the greatest thickness about 1 cm (Fig. 9). It encrusts an oyster shell that was attached to a metal wreck off Ocracoke Island. It was bright orange in life and has faded to a light buff in alcohol. It is quite firm and solid in consistency, owing to the large proportion of spicules present in this sponge. The second specimen consists of a series of upright lobes, some of which bear oscules.

The surface is superficially smooth, with abundant minute pores about 40 μ in diameter evenly distributed over the surface. At the apex of several low mounds are oscules 0.3 to 1.0 mm in diameter that may be partially closed. The surface is composed of a dense pallisade of spicules situated perpendicular to the surface. Confused masses of spicules form tracts that ascend from the interior and spread out just below the surface to form the cortical layer. The spicules are oxeas 160–260 x 4–10 μ, with occasional styles or strongyles.

This species was first described from Puerto Rico as *Petrosia halichondrioides* by Wilson (1902, p.389), and was transferred to the genus *Xestospongia* by de Laubenfels (1950a, p.51). The synonymy of this species with *Xestospongia muta* (Schmidt 1870, p.44) on the basis of color in alcohol (de Laubenfels 1953) is not accepted here. The spicules of *X. muta* are larger (340 to 420 μ in length), and its form is typically vase or cup-shaped. The presence of upright lobes in the second North Carolina specimen precludes the development of the vase or cup shape characteristic of *X. muta* or *X. calyx*.

Callyspongia vaginalis (Lamarck) Duchassaing & Michelotti — Vase sponge

Four specimens (including U.S.N.M. 23619) from the beach near Cape Hatteras, and one from Bogue Sound near Beaufort.

This species is typically composed of a series of hollow branches rising from an attached base (Fig. 12). The figured specimen is 17 cm high with the largest branch 29 mm wide. The walls of these tubular branches are about one-seventh the diameter of the branch. The sponge is lavender in life, and yellowish gray in preservatives or dry. It is toughly spongy.

The surface is roughened by large, obvious conules that project 3 to 4 mm above the surface contours. Abundant round depressions 2 mm across represent pores areas. The apical opening of the large central cavity represents the oscules in this species, and may be up to 20 mm in diameter. The dermis consists of a network of spongin fibers 30 to 180 μ in diameter with much finer fibers 7 to 16 μ in diameter forming a secondary network in the meshes of the larger fibers. An extremely thin membrane may be found across this reticulation. The internal skeleton consists of an irregular reticulation of the larger fiber. Oxeas 80–95 x 2–4 μ core the fibers.

This species was first described as *Spongia vaginalis* by Lamarck (1813, p.436), and was placed in the genus *Callyspongia* by Duchassaing and Michelotti (1864, p.56). Much of the older literature treated this species as *Spinosella sororia* (Duch. & Mich. 1864), but de Laubenfels (1950a, p.56) in reviewing this species showed that they are conspecific.

ORDER POECILOSCLERINA

The skeleton includes a reticulation of spongin fibers. Usually there is some sort of dermal skeleton, with a different type of arrangement of spicules at the surface. Microspicules are often present. This order is close to the Halichondrina in which a plumose type of fiber or spicular tract is characteristic.

Adocia tubifera (George & Wilson) Hartman

Two specimens from Beaufort, including U.S.N.M. 23610.

This sponge consists of an irregular system of cylindrical branches varying from 3 to 10 mm in

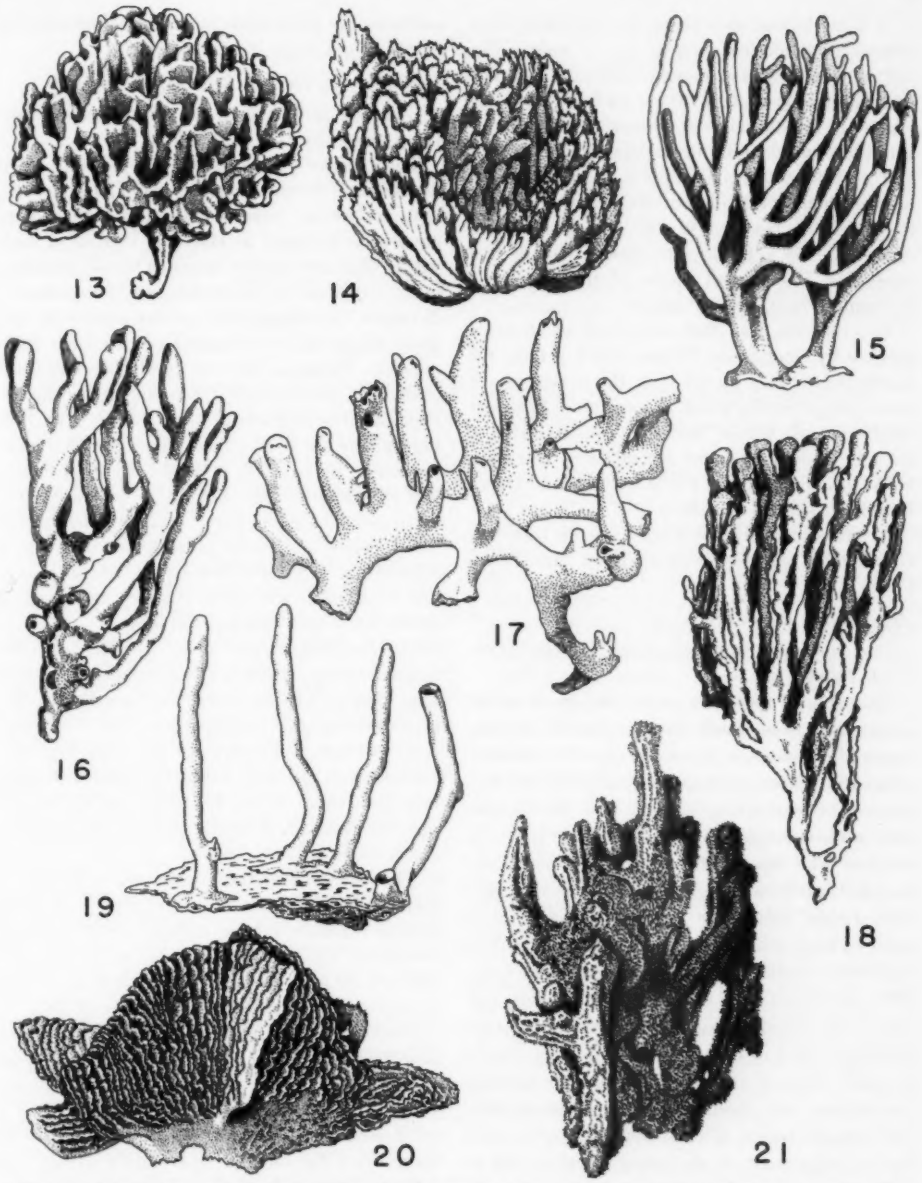


FIG. 13. *Microciona juniperina*. FIG. 14. *Hemectyon pearsei*. FIG. 15. *Endectyon tenax*. FIG. 16. *Tenaciella obliqua*. FIG. 17. *Adocia tubifera*. FIG. 18. *Microciona prolifera*. FIG. 19. *Rhizochalina oleracea*. FIG. 20. *Thalyseurypon carteri*. FIG. 21. *Dictyociona adioristica*.

diameter (Fig. 17). From these branches numerous tubes arise, 2 to 12 mm high and 2 to 4 mm wide. Their tips are thin and transparent, and surround the oscules which are 0.5 to 2.0 mm in diameter. The entire colony measures 75 mm in its greatest dimension. It is quite fragile. In life it was colored a pink or purplish tan, but this has faded to an off-white or pale yellow in alcohol.

A thin dermal membrane is supported over extensive subdermal spaces by a unispicular reticulation formed of spicules cemented together at their tips by spongin. Pores 20 to 100 μ in diameter are scattered over the surface. The endosome contains longitudinal spongin fibers, 30 to 100 μ in diameter, containing 3 to 8 spicules per cross-section. A reticulation of spicules is present in the space between fibers. The spicules are oxeas 135–175 x 4–8 μ ,with those of the dermis averaging 160 μ, and those of the endosome averaging 170 μ, slightly larger.

This species was first described as *Reneira tubifera* by George and Wilson (1919, p.145). De Laubenfels (1947) placed it in the synonymy of *Haliclona permollis*, which also occurs in Beaufort Harbor, and whose skeleton consists almost entirely of a unispicular reticulation. Hartman (1958, p.71) recognized the differences and transferred this species to *Adocia*.

Apparently *Adocia tubifera* is endemic to North Carolina; it has only been recorded from Beaufort.

Rhizochalina oleracea Schmidt

One specimen from Beaufort Harbor (U.S. N.M. 23611).

This sponge forms an encrusting layer on an oyster shell from which rise four slender, hollow, finger-like processes 1.5 to 3 mm in diameter (Fig. 19). These processes extend 30 to 35 mm above the encrusting portion and are fragile. The surface is smooth without visible pores or oscules. The tips of the slender processes are rounded and show no sign of oscular development. The dermal membrane is thin and contains a layer of tangential spicules in a reticulation. The endosome contains a series of longitudinal fibers 30 to 80 μ thick that contain many spicules, and connecting spicular tracts and scattered spicules forming a kind of reticulation. The fibers have a a small amount of spongin, and the spicular tracts have only enough to connect the spicules. The central region of each finger-like process is hollow; adjacent to it are a ring of canals 150 to 350 μ in diameter. The spicules are oxeas 94–120 x 3–6 μ.

This species was first described by Schmidt (1870, p.35). George and Wilson (1919, p.152) describes this specimen as *Phloeodictyon nodosum*. De Laubenfels (1947, p.35) recognized its identity with *Rhizochalina oleracea*. Examination of U.S. N.M. 23611, the type specimen of *P. nodosum*, confirms his identification. *Rhizochalina oleracea* is typically a West Indian species.

Lissodendoryx isodictyalis (Carter) Topsent

Common in Beaufort and Hatteras Harbors. Represented by U.S.N.M. 23630.

This species forms encrustations that often develop into irregular colonies up to 20 cm across, with numerous vertical projections. Specimens have been collected in Hatteras Harbor in midwinter that were simply lumps of tissue enmeshed in the processes of *Microciona prolifera* colonies. However, in summer this species attains an unusual degree of development on the rock jetty opposite Hatteras Harbor, producing extensive mats that coat many of the submerged rocks. Irregular vertical processes often project 2 to 3 cm beyond the encrusting mass, in exceptional cases measuring up to 10 cm in height. By their fusion, the thickness of the colony is increased and excurrent channels may become surrounded. This species is crisp in consistency. It often is colored blue-green or yellow green, but this may be due to an alga, for whole colonies and parts of others are a pale yellow or off-white.

On the surface are many slightly elevated ridges, between which is a thin dermal membrane that overlies obvious subdermal canals. In life, numerous translucent papillae extend 1 to 3 mm from the surface, but because these are very contractile, they are often withdrawn upon collection or preservation. Oscules 0.5 to 2.5 mm in diameter are often contained at the tips of vertical processes. Pores are microscopic.

The internal skeleton is primarily composed of a spicular reticulation formed of styles, while the dermis contains tylotes radially or tangentially arranged. The spicules (Fig. 15) are styles 146–190 x 4–10 μ, tylotes 178–194 x 4–6 μ, C- and S-shaped sigmas 18–40 μ, and chelas 12–28 μ.

This species was first described as *Halichondria isodictyalis* by Carter (1882, p.285) and was transferred to the genus *Lissodendoryx* by Topsent (1897, p.456). Wilson (1911, p.11) described it as *Lissodendoryx carolinensis*; and this is the name used by George and Wilson (1919) and Miner (1950). North Carolina specimens and those to the north contain larger sigmas and smaller chelas than specimens from warmer waters to the south (Hartman 1958), but this variation has not been given taxonomic status.

Mycale cecilia de Laubenfels

Many specimens from Hatteras Harbor, including U.S.N.M. 23633.

TABLE 1
Comparison of characters of Mycale cecilia *de L.*

Location	Panama	Hawaii	North Carolina
Author	de Laubenfels 1936a	de Laubenfels 1950c	Wells & Wells
Tylostyles	300 x 7–10 μ	240–250 x 4–6 μ	250–270 x 4–6 μ
Sigmas	30 μ	30–42 μ	50–58 μ
Larger chelas	22–25 μ	24 μ	30–36 μ
Smaller chelas	12–15 μ	15 μ	14–16 μ
Color	green, red embryos	pink and lavender, yellow embryos	yellow-green or yellowish tan, orange embryos

This species is represented by thin colonies up to 8 cm in diameter that encrust rocks, shells, and calcareous worm tubes. In life they are pale yellowish green or yellowish tan and quite soft and slimy. Specimens containing orange-colored embryos were collected in August 1959.

There is a thin dermal membrane containing spicules tangentially arranged, that extends over the ends of vertical polyspicular tracts. This dermis can be pulled off as a sheet, exposing dendritic patterns of subdermal canals. No oscules or pores are optically evident. Internally the sponge may be more or less cavernous, with abundant spaces. The skeleton consists of ascending spicular tracts 30 to 60 μ in diameter which branch and end in a brush of spicules at the dermis. In dry specimens, these tracts may extend beyond the dermis as a result of shrinkage of surrounding tissue. The spicules (Fig. 16) are tylostyles 250–270 x 4–6 μ, comprising the ascending tracts and scattered in the dermal membrane; sigmas 50–58 x 3–4 μ in which the axis is twisted 90°; and two size classes of palmate anisochelas, the larger 30–36 μ long and the smaller 14–16 μ long.

This species was first described from the Pacific coast of Panama by de Laubenfels (1936a, p.447). It has also been reported from Hawaii (de Laubenfels 1950a). The characters of these sponges are compared in Table 1. Sigmas and the larger class of chelas are somewhat larger in North Carolina specimens. Regarding the variations in color of this species, de Laubenfels has suggested that its color may change with age.

This is the first report of this species from the Atlantic Ocean. It is probably distributed in the West Indian region. Many other marine organisms are distributed on both sides of Central America, so this pattern of distribution is not unusual.

Xytopsues griseus (Schmidt) de Laubenfels

One specimen was collected at Beaufort by George and Wilson.

This species forms encrustations that may develop into thick massive colonies up to 8 cm long. The surface is relatively smooth. Small oscules, 1 mm in diameter, pass through a dermal layer 10 μ thick into meandering subdermal canals. Scattered spicules are found in the dermal membrane lying parallel to the sponge surface. Skeletal fibers of the endosome are largely composed of sand, with a greater representation of the proper spicules of this species present in the peripheral portions of these fibers. These fibers neither branch nor anastomose, but sometimes run to the surface at an oblique angle, presenting a branching appearance. Only inconspicuous amounts of spongin are present in the fiber. Specimens have been reported (de Laubenfels 1950a) in which algal filaments substituted for skeletal fibers. Spicules: (1) strongyles 160–180 x 2–3 μ, (2) sigmas 10–20 μ, and (3) isochelas 12–16 μ.

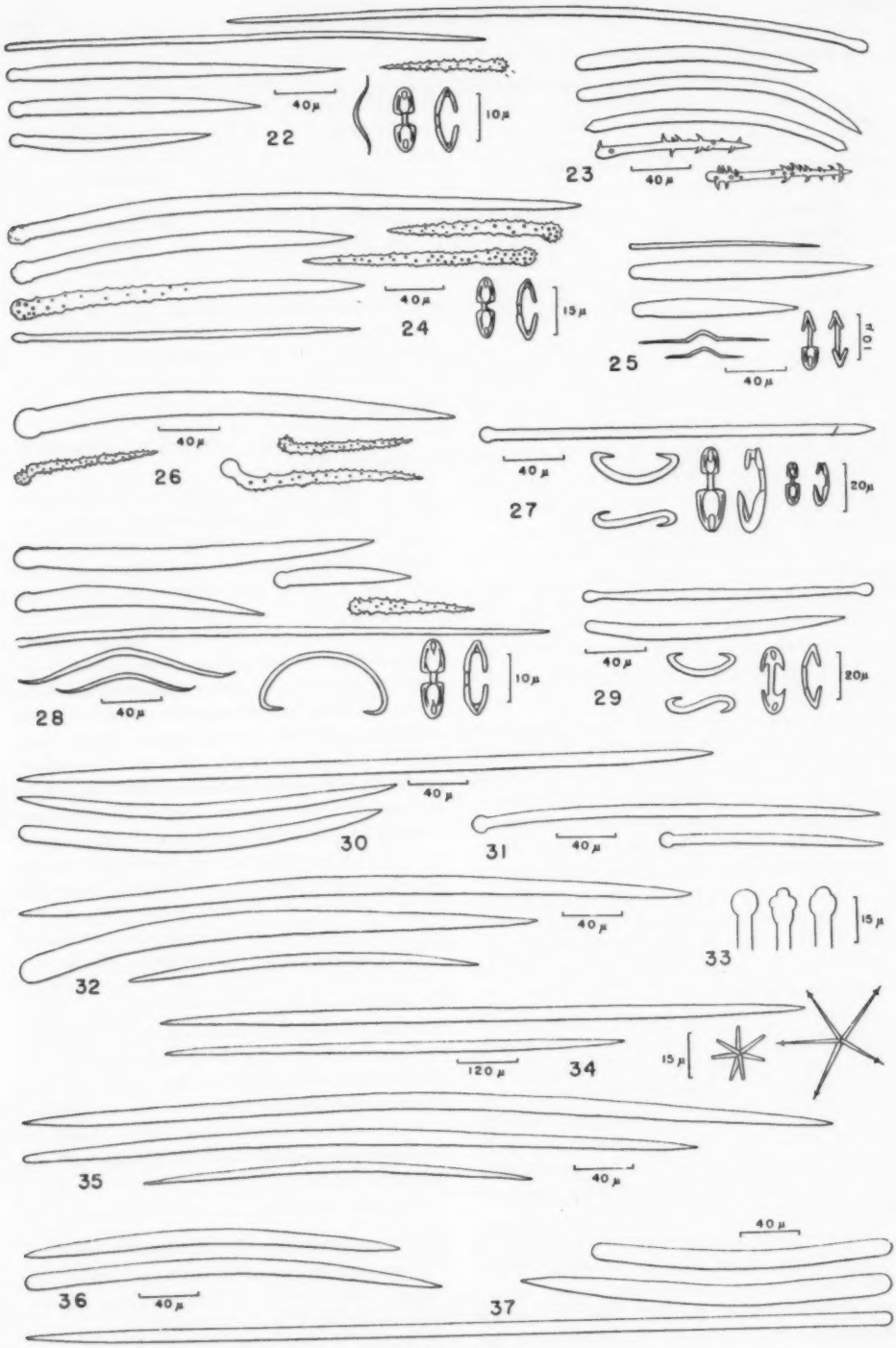
This species was first described as *Desmacidon griseum* by Schmidt (1870, p.55). It was described by George and Wilson (1919, p.154) as *Phoriospongia osburnensis*, and subsequently *osburnensis* was made the type of the genus *Xytopsues* (de Laubenfels 1936b, p.55). In 1947, de Laubenfels placed *osburnensis* in the synonymy of *Desmapsamma australis* Dendy, but later (1949a) he corrected this mistake, and synonymized it with Schmidt's species.

Xytopsues griseus is a typical West Indian form.

Microciona prolifera (Ellis & Solander) Verrill

Many specimens from Hatteras Island and Harbor, and several from Beaufort Harbor. Represented by U.S.N.M. 23618.

Colonies of this species attach to rocks, shells, pilings, algae, and alcyonarians. Initially this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40 μ
31
40 μ
32
40 μ
33
15 μ
34
120 μ
15 μ
35
40 μ
36
40 μ
37
40 μ

FIGS. 22–37

species forms a thin encrustation; many specimens from exposed situations in high salinities exhibit this form. Under typical estuarine conditions however, many erect fibers develop from this nasal membrane and form by their fusion a series of erect lobes (Fig. 18). These lobes are 2 to 5 mm in diameter and up to 20 cm high and may branch and coalesce to form a head-sized mass. The tips of these lobes are sometimes swollen and often compressed. Intermediate forms have been collected on the Hatteras Harbor jetty that exhibit an upright lamella with an irregular surface, as though it had been formed by the incomplete separation of a series of adjacent lobes.

In his description of this species in his paper on sponges of the Woods Hole area, de Laubenfels (1949a) confused it with the following species, *Microciona juniperina*. His description of growth form is primarily that of two specimens dredged off Beaufort which we have examined and found to represent the latter species. In it, a honeycomb-like structure is formed by the fusion of adjacent upright septa.

This species is bright red in life, but cold damage and other adverse conditons may produce a dull brown discoloration. Its consistency is toughly spongy.

The surface appears smooth in fresh specimens, but with drying, contraction of the flesh leaves the surface irregularly conulose. Microscopically, a thin, translucent dermal membrane extends over the ends of radial spongin fibers. Individual spicules project through this membrane, extending up to 200 μ, usually only 100 μ beyond its surface. With drying or wave erosion, this membrane and much of the soft flesh is lost, and the surface exhibits a hispid, ragged appearance due to projecting fiber ends and spicules. Oscules are not evident. In encrustations, branching fibers arise from a basal plate of spongin and project independently toward the surface. In the interior of erect lobes, there is a network of spongin fibers that leads into a peripheral zone of independent radial fibers that extend to the surface. The fibers contain many spicules, often with only their heads embedded in the spongin. Spicules (Fig. 22): (1) thick styles 150–350 x 8–14 μ, (2) thin subtylostyles 100–340 x 2–5 μ, with intermediate stages present, (3) acanthostyles 80–100 x 6–8 μ, (4) toxas 10–40 μ, and (5) chelas 12–16 μ. Toxas and chelas are rare in North Carolina material, and may be virtually impossible to find in some specimens. Their discovery is made difficult by the granular nature of the protoplasm of this species. In specimens from north of Cape Cod, toxas and chelas are more abundant and the acanthostyles are larger. The shape of the larger spicules (1, 2, and 3) may vary considerably between specimens: the swollen "tylote" endings may be well developed or absent, and the spination of the acanthostyles may also be well developed or absent.

This species was first described as *Spongia prolifera* by Ellis and Solander (1786, p. 189), and transferred to the genus *Microciona* by Verrill (in Verrill & Smith 1873, p.741). De Laubenfels (1947, 1949a) placed *Esperiopsis obliqua* George & Wilson in the synonymy of this species, and suggested that the same should be done with *Acanthella corrugata*, *Axinella acanthifera*, and *Suberites undulatus*, all four described by George and Wilson from a small number of specimens. These are discussed in this paper under *Tenaciella obliqua*, *Higginsia strigilata*, *Endectyon tenas*, and *Cliona viridis*, respectively, and are regarded as separate entities. In addition, specimens identified as *Microciona prolifera* by de Laubenfels (Pearse & Williams 1951) are included in the following species—*Microciona juniperina*.

De Laubenfels (1936b, p. 111) noted that *prolifera* is intermediate in character between the genus *Thalysias* and the genus *Microciona* as he defined them, and actually would fit *Thalysias* better than *Microciona*. However, on the basis of long usage in that combination, he left *prolifera* in *Microciona*. By his definition, *Microciona* is encrusting and has little dermal specialization, while *Thalysias* branches and has distinct dermal specialization. Because these characteristics intergrade, Vosmaer (1935) and Hartman (1955) dropped *Thalysias* into the synonomy of *Microciona*. De Laubenfels' partitioning of this group

FIGS. 22–37. Sponge spicules, drawn with the aid of a camera lucida. FIG. 22. *Microciona prolifera*. FIG. 23. *Endectyon tenax*. FIG. 24. *Dictyociona adioristica*. FIG. 25. *Tenaciella obliqua*. FIG. 26. *Hemectyon pearsei*. FIG. 27. *Mycale cecilia*. FIG. 28. *Microciona juniperina*. FIG. 29. *Lissodendoryx isodictyalis*. FIG. 30. *Pseudaxinella wilsoni*. FIG. 31. *Prosuberites microsclerus*. FIG. 32. *Teichaxinella grayi*. FIG. 33. *Terpios fugax*, heads of tylostyles showing variation in form. FIG. 34. *Dorypleres carolinensis*. FIG. 35. *Ciocalapata gibbsi*. FIG. 36. *Axinella bookhouti*. FIG. 37. *Phakellia folium*.

has merit, for it is large and widely distributed; and the existence of intermediates does not lessen the convenience of recognizing two divisions. It would appear best to recognize these two categories, using the criteria set up by de Laubenfels (1936b), but drop their rank to subgenera within the genus *Microciona*. Under this classification, *prolifera* should then be transferred to the subgenus *Thalysias*, the full designation being *Microciona* (*Thalysias*) *prolifera*.

Microciona prolifera is distributed from North Carolina northward.

Microciona juniperina (Lamarck) Hartman

Two specimens from New River fishing banks, and two specimens dredged off Beaufort Inlet. Represented by U.S.N.M. 23565.

This species produces beautiful colonies of intricate design (Fig. 13). The figured specimen is 8 cm high and 8 cm wide. From a narrow peduncle attached to a stone, arise about six branches that extend out radially and then divide and form a complicated network of branching and anastomosing septa directed toward the surface. The peduncle is 1 cm tall and 10 by 4 mm thick. The septa are 2 to 3 mm thick, and enclose cylindrical channels 5 to 10 mm in diameter that are open both above and below. Toward the periphery of the colony the development of septa is less pronounced; here, flat, lumpy branches, not yet fused with one another, present a more irregular appearance. Even in the central part of the network, the upper margins of the septa are not evenly developed.

The sponge is red, and tough in consistency. The surface is more compact and even than that of *Microciona prolifera*. There is a thin derma membrane pierced by many hispidating spicules. Minute pores are visible on the sides of septa, and stellate patterns mark the location of oscules. The thin dermal membrane is easily lost, exposing the many dermal brushes of projecting spicules. A reticulation of spongin fibers forms the interior skeleton. Spicules (Fig. 28): (1) stout subtylostyles 210–310 x 13–18 μ, characteristic of the surface hispidation and the fibers; (2) slender subtylostyles 120–405 x 2–5 μ, characteristic of the dermal membrane; (3) short subtylostyles, often with spiny heads, 74–114 x 8–14 μ, echinating the fibers; (4) toxas 50–144 x 4 μ, (5) very rare sigmas 76–110 μ, and (6) palmate chelas 16–20 μ. Apparently, the occurrence of specimens containing sigmas is unusual, for this is only the second report of their occurrence in this species, the first being that of Schmidt (1870, p.56, as *Tenacia clathrata*). As in *M. prolifera*, the spination of the short echinating spicules is variable.

On the surface of *Microciona juniperina* specimens from New River fishing banks were a number of colonial sea-anemones that have been identified as the zoanthid *Parazoanthus swiftii* (Duch. & Mich. 1861) by Dr. C. E. Cuttress of the U.S. National Museum.

The unusual form of this sponge, the striking contrast between the thickness of its stout subtylostyles and slender subtylostyles, and the large size of its toxas readily distinguish this sponge from *Microciona prolifera*.

This species was originally described by Lamarck (1813, p. 444), and redescribed by Topsent (1932a, p. 97). Hartman (1955, p. 171) has transferred it to the genus *Microciona* and given an extensive list of synonyms. De Laubenfels (1936b) established *juniperina* as the type species of *Thalysias*. In accordance with Hartman's placing this species in *Microciona*, *Thalysias* is dropped to a subgenus, and the full designation for this species becomes *Microciona* (*Thalysias*) *juniperina*.

There has been some question raised about the correct spelling of the generic name *Thalysias* Duchassaing & Michelotti, 1864 (or subgeneric name under the present classification). Carter (1882) and Hartman (1955) point out that through printer's errors four different spellings of this name appear in Duchassaing and Michelotti's paper. However, there is no question as to which is the correct spelling, for three misspellings are corrected to *Thalysias* on pages 122 and 123 of the original publication, and only that spelling remains. It seems possible that their four-page "Errata" was not bound with all copies of this publication, for it is variously listed with 115, 119, or 124 pages in the literature citations of recent toxanomic works. A copy of Duchassaing and Michelotti (1864) in the United States Library of Congress, Washington, D. C., contains an "Errata" on pages 121 to 124 bound into the volume.

In 1946, de Laubenfels dredged two specimens from 5 meters depth off Beaufort that he regarded as *Microciona prolifera* "in its most extremely proliferated form." He later described these specimens as the "extreme perfection" of *M. polifera*, in his paper on the sponges of Woods Hole (de Laubenfels 1949a). We have examined these and shown one in Fig. 13. They are typical specimens of *Microciona juniperina*. A similar situation ex-

ists with the specimens from New River fishing banks. These specimens, which had been labeled by de Laubenfels as "abnormal, dubious *Microciona prolifera*", and which were recorded by Pearse and Williams (1951), are also *M. juniperina*.

This species is typical of the West Indian region.

Thalyseurypon carteri (Topsent) de Laubenfels

Two specimens from New River fishing banks, represented by U.S. N.M. 23656.

This species forms this fan-like laminae whose surfaces are corrugated by ridges (Fig. 20). The figured specimen is 55 mm long and 40 mm high, and the main lamina is 3 to 10 mm thick. Lateral laminae project from the main lamina like buttresses. In alcohol this species is brownish gray and spongy. The surface is prominently marked by thin ridges that extend across the main axis of the sponge. At the surface, these ridges form a divergent, branching pattern. Between these ridges, a depressed, fleshy dermis extends across the intervening valleys which contain oscules 0.5 to 1.0 mm in diameter. The protruding ridges are made hispid by brushes or projecting spicules.

Internally, the skeleton consists of a reticulation of radial multispicular spongin fibers 30 to 80 μ in diameter and connecting transverse fibers, usually containing only one spicule. The radial fibers may be surrounded by abundant spicules that have their heads embedded in spongin. Spicules: abundant acanthotylostyles 150–350 x 8–15 μ with very small spines, characteristic of the interior tracts; and smooth tylostyles 250–900 x 10–14 μ, characteristic of the surface brushes.

This species was first described as *Clathria carteri* by Topsent (1889, p. 38), and transferred to the genus *Thalyseurypon* by de Laubenfels (1936b, p. 107). The latter author noted that this species is identical with *C. foliacea* described by Topsent on the next page, and he used *foliacea* as the specific name. Because *carteri* was applied to the species first, *Thalyseurypon carteri* is the valid name for this species, and *foliacea* falls to its synonymy. This species was identified as *Thalyseurypon foliacea* in the New River collection and reported that way by Pearse and Williams (1951). An examination of the specimens from the New River collection that were labeled *Thalyseurypon carteri* shows that they are actually *Homaxinella waltonsmithi*.

This is typically a West Indian species.

Eurypon clavata (Bowerbank) Gray

One specimen from off Ocracoke Inlet, U.S. N.M. 23632.

This species is represented by a thin encrustation of less than 1 cm² in area that was attached to a conglomerate rock dredged from 10 meters. It was orange and fleshy in life. The surface is marked by a projecting hispidation of scattered individual spicules. No oscules or pores can be observed. The entire sponge is less than 1 mm in thickness. Its spicules are extremely long tylostyles 1000–2000 x 15–16 μ and abundant short acanthotylostyles 70–205 x 5–11 μ. The tylostyles stand erect upon the substrate and are echinated by the smaller spiny spicules, as though they were the spongin fibers of other sponges in this family. The spines on the acanthotylostyles may be quite obvious.

This species was first described as *Hymeraphia clavata* by Bowerbank (1866, p. 143), and transferred to the genus *Eurypon* by Gray (1867, p. 521). It is common on the coast of Europe and has been recorded at Bermuda (de Laubenfels 1950a).

Dictyociona adioristica de Laubenfels

Many specimens collected on Cape Hatteras beach, represented by U.S.N.M. 23623, and one specimen from Beaufort Harbor.

This sponge attaches to hard substrates such as shells, alcyonarian colonies, conglomerate rock, and calcareous worm tubes. Its colonies are composed of a series of irregularly shaped upright processes that often branch and coalesce (Fig. 21). The figured specimen is 7 cm high. Individual branches range from 3 to 10 mm in width. When fresh, this species was bluish gray and toughly spongy.

The surface contains a thin dermal membrane that is easily lost. No oscules or pores are visible. The internal skeleton includes a reticulation of spongin fibers that give rise to a zone of loosely matted, branching radial fibers. The spongin of these fibers is unusually firm and resistant to bases. At the surfaces, these fibers branch at wide angles and end in brushes of erect spicules. Where the dermis has been lost, these spicular brushes give this sponge a fuzzy appearance. Spicules (Fig. 24): (1) subtylostyles with spiny or lumpy heads 150–350 x 8–14 μ common in fibers and surface brushes, (2) acanthotylostyles 150–200 x 6–8 μ in fibers, (3) acanthotylostyles 70–100 x 3–6 μ echinating the fibers, (4) slender

tylostyles 210–250 x 2–4 μ in the dermis, and (5) chelas 16–20 μ.

This species was first described from Florida by de Laubenfels (1953, p. 526).

Endectyon tenax (Schmidt) Topsent

Many specimens from the Hatteras area including U.S.N.M. 23616, and two specimens from New River fishing banks.

This species usually forms erect colonies up to 16 cm high that produce many slender branches (Fig. 15), although colonies may occur with flattened lobes. The figured specimen is 9 cm high. The basal stalk may be up to 17 mm wide, but the many branches that it gives rise to are only 2 to 4 mm in diameter. The pattern of branching may be palmate or trichotomous, as well as dichotomous; commonly branches fuse where they come into contact. One specimen from the New River collection has several flattened lobes 1.5 to 3 mm in thickness arising from a narrow attachment. This species is red colored in life, and yellowish gray in dried specimens. It is quite stiff and will break when dry rather than bend.

The surface is optically smooth, with stellate patterns of narrow radiating grooves marking the location of oscules. Pores and oscules are microscopic. Under a lens the surface is hispid and punctate. Internally, there is a firm axial core that contains longitudinal spongin fibers. The peripheral zone contains radial fibers (1 or 2 spicules per cross-section). At the ends of radial fibers several spicules stand erect, points directed outward. Spicules (Fig. 23): (1) numerous styles 118–230 x 4–14 μ, (2) thin subtylostyles 170–400 x 4–8 μ, and (3) acanthostyles 85–130 x 3–10 μ with recurved spines 3 to 8 μ long. One New River specimen contains many abnormal spicules that have enlarged central canals and step-wise ends. There were also some abnormal spicules with step-wise points at each end, oxeas 130–190 x 6–10 μ, one of which is shown in Figure 26.

George and Wilson (1919, p. 159) described a specimen of this species from the Beaufort area as *Axinella acanthifera*, and it is treated by Miner (1950) under that name. That sponge contained flattened lobes 2 to 3 mm thick, resembling one of the New River specimens. De Laubenfels (1947, 1949a) suggested that it was a malformation of *Microciona prolifera*, but the spiculation easily sets this species apart from that and other species.

This species was first described as *Phakellia tenax* by Schmidt (1870, p. 62), and was transferred to the genus *Endectyon* by Topsent (1920, p. 308). It is a West Indian species.

Hemectyon pearsei, new species

One specimen (type: U.S.N.M. 23651) dredged from 5 meters depth off Beaufort.

This sponge is a small ellipsoidal mass 30 mm long and 20 mm high composed of many compressed, vertically elongated units (Fig. 14). It was attached to a piece of conglomerate rock. Its consistency is stiffly spongy. The surface is marked by many upright ridges 1 to 1.5 mm apart from which spicules project. Between these ridges, a smooth dermal membrane covers deep subdermal spaces. This membrane is pierced by many circular openings 150 to 600 μ in diameter and several accidental tears. Presumably the larger openings are oscules, and the smaller ones pores, though separation into two size classes is difficult. The interior skeleton consists of a reticulation of vertical spongin fibers and radial fibers. The spicules (Fig. 26) are smooth subtylostyles 190–340 x 10–14 μ, characteristic of the dermal brushes and fibers, and small acanthostyles 85–115 x 3–7 μ that are sharply bent near their heads.

This species is named in honor of the late Dr. A. S. Pearse, founder of the Duke University Marine Laboratory at Beaufort.

This specimen was one of two similar sponges collected by de Laubenfels in 1946, and identified as *Higginsia coralloides*, now known as *Higginsia strigilata*. The second sponge was correctly identified. Although there is much similarity in the growth form of the two sponges, the presence of spiny tylostyles in *Hemectyon* and spiny oxeas in *Higginsia* serve to distinguish them.

The growth form of this species resembles that of *Hemectyon hyle* de Laubenfels (1930, p. 28) of the Pacific Coast, as pictured by Dickinson (1945, Fig. 64), but the echinating acanthostyles of that species are much larger (180 to 320 μ long vs. 85 to 115 in *pearsei*). This species is separated from *Hemectyon hamata* (Schmidt 1870, p. 62) of the West Indies, by a similar difference in size of the acanthostyles and other features of its spiculation.

Tenaciella obliqua (George & Wilson), new combination

Many specimens from Cape Hatteras beach, two from Beaufort area, and one from New River fishing banks. Represented by U.S.N.M. 23635, 23613, and 23658.

This species begins as an encrustation on submerged objects such as alcyonarian colonies, pelecypod shells, or algae. It later develops a series of erect finger-like lobes that often extend 4 to 9 cm, rarely up to 15 cm, beyond its attachment. The majority of specimens from Hatteras encrust colonies of the alcyonarian *Telesto*, leaving obvious openings up to 1.3 mm at the tips of the alcyonarian branches, through which the polyps would extend in life (Fig. 16). The alcyonarian skeleton may often be seen through these openings. Above this encrusting base rise finger-like lobes 3 to 5 mm in diameter that may branch or be somewhat lumpy.

In life, this species is orangish red or bright red, but this color quickly fades with death, passing through a light orange stage, to an eventual yellowish gray in dried specimens. Beach specimens may bleach to off-white. The consistency is typically spongy-flexible and compressible.

The surface is even and finely punctate, with minute pores penetrating the thin dermal membrane. Oscules about 1 mm in diameter may be scattered over the surface. The interior skeleton consists of obliquely radial spicular fibers joined by connecting fibers. The spicules are contained within the spongin of these fibers. Spicules (Fig. 25): (1) abundant styles 110–150 x 6–10 μ tapering at both ends and sometimes spiny, (2) thin tylostyles 120–140 x 2–4 μ. (3) toxas 20–60 μ, and (4) chelas 9–11 μ with a twisted axis. The toxas and chelas may be rare in some specimens.

After storms, hundreds of specimens of this species have been collected on beaches in the Cape Hatteras area. Many of these have been fresh specimens. These have been compared with specimens collected by George and Wilson in the Beaufort area, including the type specimen (U.S.N.M. 23612), and they agree in every detail. They are evidently common on hard bottoms offshore.

This species was first described by George and Wilson (1919, p. 148) as *Esperiopsis obliqua*, and is treated by Miner (1950) under that name. The fact that the spicules of *obliqua* are enclosed in the spongin fibers and not in an echinating position places this species in the genus *Tenaciella* of the family Amphilectidae rather than in the genus *Esperiopsis* of the Ophlitaspongiidae, as defined by de Laubenfels (1936b). De Laubenfels (1947, 1949a) placed this species in synonymy with *Microciona prolifera*. In the collection of sponges from New River fishing bank is a specimen of this species identified by de Laubenfels as *Haliclona oculata* which is the basis of that record published by Pearse and Williams (1951). It bears a superficial resemblance to southern representatives of that species (see Hartman 1958) as do most colonies of *Tenaciella obliqua*. An examination of its spiculation shows the presence of styles rather than the oxeas typical of *Haliclona oculata*. It seems likely that the records of *Haliclona oculata* listed for North Carolina by Hartman (1958) probably are of *Tenaciella obliqua*. These are Fort Macon, by Coues and Yarrow (1879, p. 312, as *Chalina arbuscula*), and North Carolina, by Verrill (in Verrill & Smith 1873, p. 743, as *Chalina arbuscula*).

Tenaciella obliqua is distributed from North Carolina southward.

ORDER HALICHONDRINA

The sponges in this order are either marked by a distinctive, plumose architecture (as contrasted with the reticulate pattern found in the Poecilosclerina) or the development of a dermis that contains many tangential spicules. The spicules are usually oxeas, styles, or strongyles. Tylostyles and spiny spicules are absent in the forms represented, except the occurrence of spiny oxeas in *Higginsia*. Microspicules are rare, being represented by raphids in a few forms and the spiny oxeas of *Higginsia*. Because the two orders are very close, the separation of the Halichondrina from the Poecilosclerina has been questioned by de Laubenfels (1953).

Axinella bookhouti, new species

One specimen (type: U.S.N.M. 23659) from New River fishing banks.

This sponge consists of a series of upright lobes rising from a hard substrate (Fig. 39). Each lobe produces flattened branches that fuse with adjacent branches from other lobes, forming poorly defined lamellae. The entire colony is 5 cm high and 6 cm wide, with individual upright lobes being 6 to 10 mm wide, and oval rather than circular in cross-section. The lamella formed by the fusion of branches is about 3 mm thick near its margin. This sponge is light brown and stiffly spongy in alcohol.

The surface is finely punctate and microscopically hispid from projecting spicules. Stellate patterns of radiating grooves mark the location of oscules. These grooves are normally covered by a thin dermal membrane that is easily lost, the oscules being the openings 0.7 to 1.3 mm in diameter in this dermal membrane at the center of the radiating pattern. The subdermal grooves

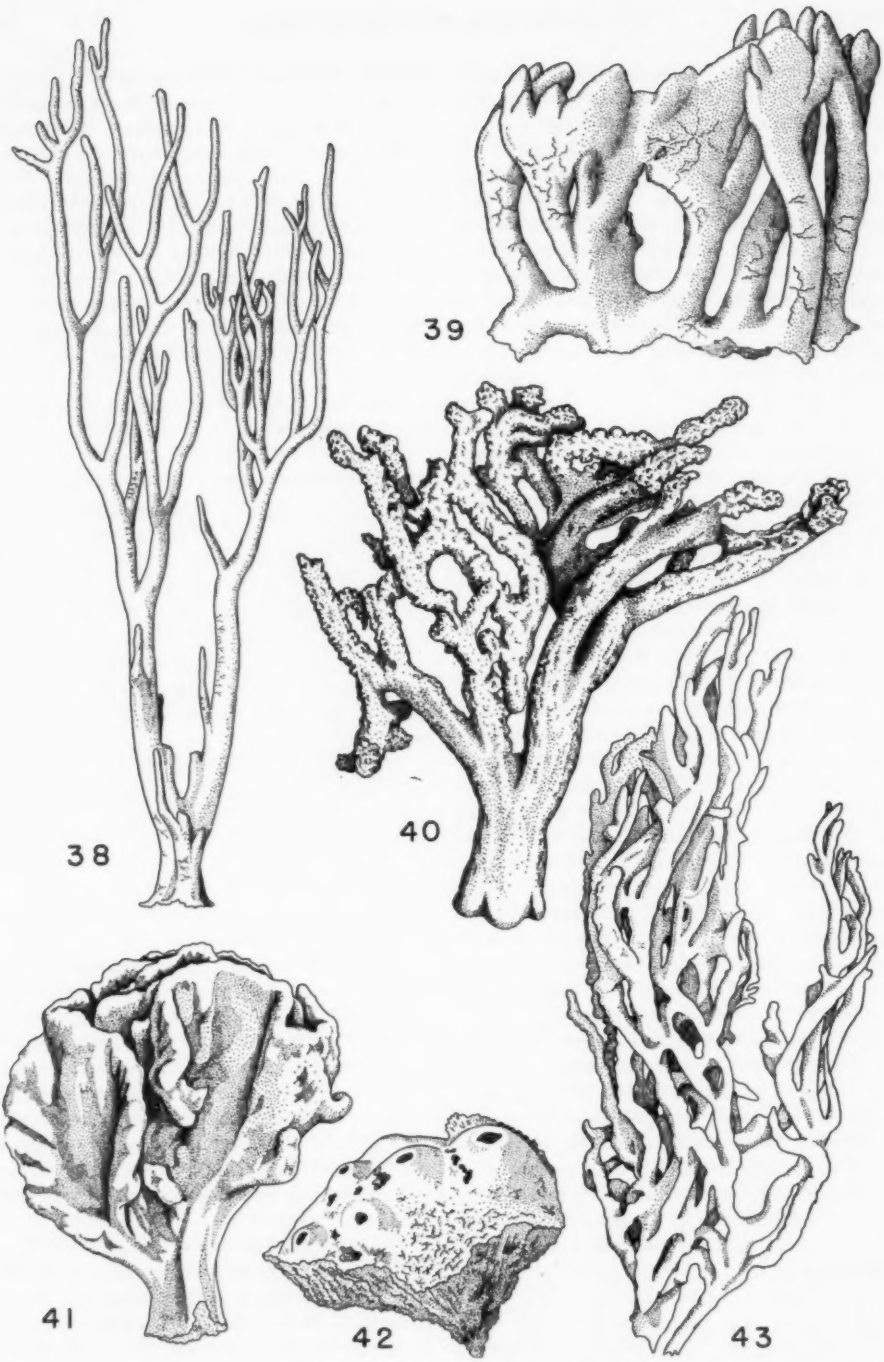

FIG. 38. *Homaxinella rudis*. FIG. 39. *Axinella bookhouti*. FIG. 40. *Axinella reticulata*. FIG. 41. *Homaxinella waltonsmithi*. FIG. 42. *Pseudaxinella rosacea*. FIG. 43. *Axinella polycapella*.

average about 0.8 mm in width. The internal skeleton includes a stiff central core that consists of closely packed vertical fibers. In the peripheral zone are radial spicular tracts 50 to 80 μ wide, containing considerable spongin as well as spicules projecting at all angles. At the surface, these tracts widen, are joined by connectives, and end in a few projecting spicules. The spicules (Fig. 36) are all somewhat curved: (1) stout styles 190–280 x 8–12 μ, comprising the bulk of the spiculation; (2) thin styles 280–480 x 5–9 μ, characteristic of the dermal hispidation; and (3) less abundant oxeas 220–240 x 9–12 μ, more common in the interior.

This specimen bore de Laubenfel's label marked *"Ophlitaspongia species novum,"* but the presence of oxeas and the absence of toxas exclude it from that genus. Instead, its features place this sponge in the genus *Axinella*.

This species is named in honor of Dr. C. G. Bookhout, director of the Duke Marine Laboratory.

Of the species of *Axinella* from the West Indian region, *A. clava* Schmidt possesses a different growth form and *A. walpersii* (Duchassaing & Michelotti), different surface features. Both are very inadequately described. The spiculation of *A. nanaspiculata* Hartman and *A. polycapella* de Laubenfels consists principally of oxeas with very few styles, whereas *bookhouti* contains abundant styles. *Axinella reticulata* Ridley & Dendy possesses very different surface characteristics; and in addition, most of these species have significantly different spicule sizes and different forms.

Axinella polycapella de Laubenfels

One specimen from Beaufort and four from the Hatteras area, including U.S.N.M. 23621.

This species forms tall, irregularly branching colonies (Fig. 43). The figured specimen is 70 cm high. The branches are 18 to 25 mm with a hard central core 5 to 14 mm in diameter. The base of a colony may be 50 mm in its greatest dimension, and although the main stalk and major branches taper gradually, the branch tips are still 12 to 18 mm across and stubby. The cortical regions of two beach-worn specimens have been eroded, presenting a very different appearance; only the central core remains, in which the tips of the branches are pointed. This species is reddish in color and stiffly spongy in consistency.

The surface is coarsely punctate, with scattered oscules 1 to 2 mm across, and many pores about 1 mm in diameter. The internal skeleton includes a dense central core containing longitudinal fibers and obliquely radial spicular tracts 40 to 80 μ across, which branch and form an irregular reticulation with many connectives near the surface. Spicules often project from these tracts at a variety of angles. The spicules are oxeas 180–250 x 5–10 μ, occasional ones as large as 320 x 12 μ. Occasional spicules occur with one or both ends rounded.

This species was first described from Florida by de Laubenfels (1953, p. 530).

Axinella reticulata Ridley & Dendy

One specimen (U.S.N.M. 23625) dredged off Cape Hatteras, and two from the Cape Hatteras beach.

This species forms erect, branching colonies in which the branches may fuse where they come into contact (Fig. 40). The figured specimen is 33 cm high, and its base is 5 cm wide. The branches narrow to a diameter of about 12 mm. It was red-orange in color and stiffly spongy in life.

The surface has a coarsely irregular reticulation and scattered spicules in the dermal membrane, plus many depressions. Oscules lead into deep canals 1.7 to 1.0 mm in diameter, and abundant pores 400 to 900 μ perforate the dermal membrane. The thin dermal membrane forms small conules over projecting fiber ends, but is easily destroyed. Internally, there is a dense axial core of closely packed longitudinal fibers from which plumose polyspicular tracts 50 to 80 μ radiate obliquely toward the surface. There are many connectives and scattered spicules. Spicules: oxeas 280–380 x 8–22 μ bent at their midpoint, and less abundant styles 240–300 x 14–18 μ.

This species was first described by Ridley and Dendy (1887, p. 184). It is found in the West Indian region.

Homaxinella rudis (Verrill) de Laubenfels

Many specimens from the Cape Hatteras beach, including U.S.N.M. 23620, and one specimen from Fort Macon beach in the Beaufort area.

This species produces graceful branching colonies up to 70 cm high, with gradually tapering branches 5 to 14 mm in diameter (Fig. 38). The figured specimen is 50 cm high and has a basal stalk 3 cm in diameter that was attached to rock. Fresh specimens are reddish and moderately flexible. The surface is superficially smooth, but close examination reveals many small pores evenly dis-

FIG.

tributed over the surface. A stellate pattern of grooves radiates from each oscule, which is less than 1 mm in diameter. The thin dermis is easily lost, exposing projecting spicules and fiber ends that make the underlying layer microscopically hispid. The internal skeleton contains a dense central axis in which fibers containing much spongin run vertically. In the peripheral zone, plumose spicular tracts 30 to 50 μ in diameter radiate to the surface. Unispicular connective fibers may join these radial tracts. The spicules are curved styles 200–300 x 5–10 μ.

This species was first described from Bermuda as *Axinella rudis* Verrill (1907, p. 341), and was transferred to the genus *Homaxinella* by de Laubenfels (1950a, p. 87). In 1950, de Laubenfels also placed *Axinella apressa* Verrill (1907, p. 340) in synonymy with this species; but this is questionable.

In both Verrill's description and de Laubenfel's description of specimens from Bermuda (1950a), a peculiarly roughened surface was noted on the branches. However, these North Carolina specimens and Florida specimens described by de Laubenfels (1953) possess a smooth surface. This difference can be attributed to differences in water movement in the natural habitat. At Bermuda, de Laubenfels found this species only in enclosed bodies of water, and it can be assumed that Verrill's specimens came from the same area, where the species is abundant. In contrast, North Carolina and Florida specimens came from open, coastal areas with a greater degree of water movement. This correlation fits the generalization that oscules and oscular chimneys are better developed where there is less water movement.

This species is distributed from North Carolina southward.

Homaxinella waltonsmithi de Laubenfels

Two specimens from New River fishing banks, represented by U.S.N.M. 23660.

This species forms beautiful colonies composed of intricate, branching folds of sponge tissue that arise from a stout basal stalk (Fig. 41). The figured specimen is 9 cm high and 7.5 cm wide. Its basal stalk is 1 to 2 cm in diameter and the blade is 3 to 6 mm thick. This species is bright orange-red in life, and spongy in consistency.

Although optically smooth, the surface is slightly hispid and bears many microscopic tubercles which are formed of the enlarged ends of radial spicular tracts. Small, evenly-distributed pores penetrate this surface plush. Over grooves between the folds, oscules 0.5 to 1 mm occur in the dermal membrane, leading to vertical subdermal canals. Oscules also occur at the upper margin of the fold, often marked by a few radiating grooves that extend from the blade surface. Internally, there is a firm axial region that contains longitudinal spicular tracts, and a peripheral zone in which the tracts are perpendicular to the surface. These radial tracts contain much flesh and spongin and are joined by many connectives. They become thicker near the surface. The spicules are styles 210–340 x 9–16 μ, many sharply bent.

This species was originally described by de Laubenfels (1953, p. 533) from a single specimen collected in the Gulf of Mexico off Florida. His specimen differed from these in having no visible pores or oscules; presumably they had contracted at the time of collection. These specimens from the New River fishing banks had been labelled by de Laubenfels as *Thalyseurypon carteri* and were recorded as such by Pearse and Williams (1951).

Pseudaxinella rosacea (Verrill) de Laubenfels

Two specimens from New River fishing banks, represented by U.S.N.M. 23661.

This species may exhibit either of two growth forms: a semi-encrusting mass up to 5 cm thick (Fig. 42) or an upright, irregularly lobate cylinder up to 7 cm high. Both forms are represented. In life, this species is orange-red and spongy in consistency.

The surface is composed of a plush of many narrow, obliquely radial columns that end at the surface with a tuft of erect spicules. Many small pores are evenly distributed over the surface between columns. Much of the surface is coarsely roughened by these columns, but they are more closely packed at the tips of lobes and around oscules. Oscules are often conspicuous, 3 to 4 mm in diameter, and usually located on slight elevations. Internally, there is no axial specialization. The columns contain confused masses of spicules: oxeas 300–400 x 5–8 μ in the core, and curved styles 220–400 x 8–11 μ projecting from their sides.

This species was first described as *Axinella rosacea* by Verrill (1907, p. 341), and was transferred to the genus *Pseudaxinella* by de Laubenfels (1949b). It is distributed from North Carolina southward.

Pseudaxinella wilsoni, new species

Two specimens from New River fishing banks, including the type: U.S.N.M. 23662.

The type specimen consists of an upright, hand-shaped plate rising from a narrow basal stalk (Fig. 48). Evidently this plate is the result of fusion or incomplete separation of a series of vertical processes, for individual lobes are still discernible at the margin of the colony. It is 7 cm high and 4.5 cm wide; the plate is 3 to 5 mm thick. It is reddish in color and toughly spongy in alcohol. The surface is minutely punctate and hispid with projecting spicules. Deeply incised grooves 2 to 4 mm long form a pattern that radiates from the basal attachment; they probably represent oscules. In the not-yet-fused upright lobes, stellate patterns of radiating grooves mark the location of oscules. The interior is composed of closely packed plumose spicular tracts. There is no axial specialization. The spicules (Fig. 30) are styles 180–230 x 7–10 μ and oxeas 210–360 x 6–8 μ.

This species is named in honor of the late Dr. H. V. Wilson, who pioneered in the study of North Carolina marine sponges.

The absence of a firm axial core in *wilsoni* places this species in the genus *Pseudaxinella*. The unusual growth form of this species and differences in oscules and other surface features separate *wilsoni* from *Pseudaxinella rosacea*.

Teichaxinella grayi, new species

Four specimens from New River fishing banks, including the type: U.S.N.M. 23663.

This species forms semi-encrusting masses whose surface is strikingly marked by deep grooves. The type specimen (Fig. 45) is 5 cm long, 3 cm wide, and 1.5 cm thick. In alcohol, it is pale straw-colored and stiffly spongy. The surface is coarsely porous. Stellate patterns of deep grooves radiate from oscules, dividing the surface into polygonal areas where one system of grooves meets another. The grooves may be 2 to 3 mm deep and 1 to 2 mm wide; the oscules, about 1 mm in diameter, are found in the deepest part of the grooves. The interior is composed of plumose spicular tracts that form radial columns at the surface. These columnar units are unconnected beyond a point 1.5 to 3 mm from the surface. At the surface the columns end in a group of projecting spicules. There is no axial specialization. The spicules (Fig. 32) are abundant styles 240–300 x 9–15 μ, oxeas 240–300 x 11–14 μ, and oxeas 360–460 x 5–10 μ.

This species is named in honor of Dr. I. E. Gray, Professor of Zoology, Duke University. It is placed in the genus *Teichaxinella* as defined by de Laubenfels (1936b), because this genus contains sponges in which the growth form precludes the possibility of the axial specialization typical of the Axinellidae. The other species placed in this genus by de Laubenfels possess a thin lamellate shape. The type species, *T. shoemakeri* de Laubenfels (1936b, p. 129), posesses much longer styles, about 500 to at least 1200 μ long, as well as an erect lamellate form.

Phakellia folium Schmidt

Two specimens from New River fishing banks, represented by U.S.N.M. 23664.

This species forms beautiful fan-shaped colonies arising from a short, stout base (Fig. 44). The figured specimen is 12 cm high and 11 cm wide, with a base about 2 cm in diameter. The compressed blade is 2 to 3 mm in thickness and supported by several radiating ribs that are similarly compressed. The margin is undulating and somewhat lobed. In alcohol, this species is chocolate brown, its consistency is stiffly spongy.

The surface is minutely hispid. Pores about 100 μ in diameter are evenly distributed in the dermal membrane. On the stalk and on the margins of the blade and supporting ribs, dendritic patterns of radiating grooves mark the location of oscules about 300 μ in diameter. Internally, the skeleton contains closely packed vertical fibers that give rise to tracts that arch out toward the surface, branch, and are joined by many connectives. Spicules (Fig. 37): (1) very abundant styles 200–320 x 8–13 μ, bent about one-third the distance from their head end, (2) rare strongyles 148–195 x 5–8 μ, and (3) long styles 400–600 x 7–8 μ, characteristic of the surface hispidation.

This species was first described by Schmidt (1870, p. 62) from Florida. The original description left much to be desired (once de Laubenfels regarded it as "unrecognizable"), and no further description has been published. The collection of these specimens by Pearse and Williams (1951) has permitted us to illustrate the spicules and the distinctive growth form of this species, and provide a more complete description.

Higginsia strigilata (Lamarck) de Laubenfels

Two specimens from Hatteras Island, one from off Beaufort Inlet, and one from New River fishing banks (U.S.N.M. 23665).

Colonies of this species produce broad, curving lobes that rise from a narrowed base (Fig. 52). These lobed processes may fuse with one another producing a curved or folded lamella with radial

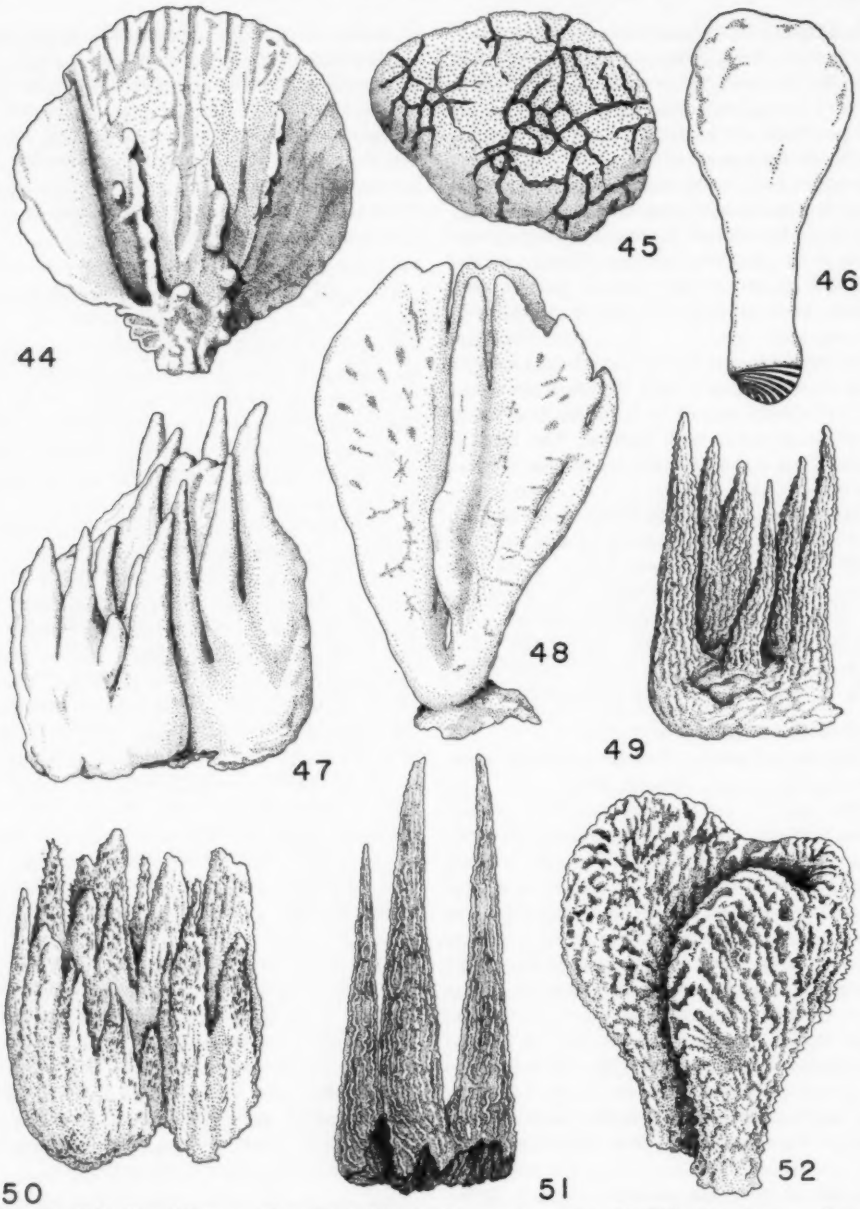

FIG. 44. *Phakellia folium.* FIG. 45. *Teichaxinella grayi.* FIG. 46. *Suberites ficus.* FIG. 47. *Ciocalapata gibbsi.* FIG. 48. *Pseudaxinella wilsoni.* FIG. 49. *Ciocalypta penicillus.* FIG. 50. *Hymeniacidon heliophila.* FIG. 51. *Oxeostilon burtoni.* FIG. 52. *Higginsia strigilata.*

corrugations on their faces. Radial ridges run across the margin of the lamella, giving rise to rows of tufts or elongated conules of tissue, 2 to 3 mm long. The lamella plus its corrugations may be up to 15 mm in thickness. This species is orange-red in life, but fades to a yellowish gray upon drying. It is stout or firm in consistency.

A dermal membrane about 30 μ thick overlies many of the interconular valleys, with irregular apertures leading into subdermal spaces up to 4 mm deep. The surface of the fleshy tufts is hispid with projecting spicules. In the interior is a system of branching and anastomosing vertical fibers that contain much spongin. In peripheral regions, oblique radial spicular bundles widen and extend to the surface where they form the surface tufts of oscules. The spicules are (1) abundant oxeas 400–600 x 16–28 μ, bent at the mid-point, (2) less abundant styles and strongyle modifications 250–470 x 20–25 μ, (3) smaller oxeas 300–500 x 4–8 μ, possibly growth stages of (1), (4) long narrow styles 640–960 x 8–15 μ characteristic of the surface hispidation, and (5) acanthoxeas 70–160 x 3–4 μ with spines up to 3 μ long, echinating the fibers and filling much of the surface tufts.

This species was first described as *Spongia strigilata* by Lamarck (1813, p. 450) and was transferred to the genus *Higginsia* by de Laubenfels (1953, p. 534). George and Wilson (1919, p. 161) described a sponge from Beaufort as *Acanthella corrugata*. In 1936, de Laubenfels transferred *corrugata* to the genus *Dragmaxia*; then he later (1947, 1949a) suggested that *corrugata* might be a malformation of *Microciona prolifera*. Except for the omission of acanthoxeas, their description of *corrugata* resembles one of these Hatteras specimens. It is well recognized that microspicules may not be present or may be very rare in specimens of a species that typically possesses them. It seems most likely that George and Wilson had a specimen of *H. strigilata* that lacked acanthoxeas. De Laubenfels (1947) reported this species from Beaufort as *Higginsia coralloides* Higgin 1877, and that name has been used in much of the older taxonomic literature. De Laubenfels also identified a specimen from New River fishing banks as *Myrmekioderma* n. sp. (Pearse & Williams 1951). This species is typical of the West Indian region.

Halichondria bowerbanki Burton

Many specimens from Hatteras and Ocracoke Harbors, several on Cape Point, and two from Beaufort, including U.S.N.M. 22724.

Colonies of this sponge assume a variety of shapes, starting as low encrustations, and often developing into a mass of erect lamellate ridges, or occasionally a series of rounded anastomosing branches. The largest colony observed was 30 cm wide. The sponge is typically a straw-yellow or beige color, sometimes a pale orange. It is soft and easily torn. Dried specimens are white and stiff.

The surface is irregular, with low ridges and conules extending above a thin dermis. This dermis contains narrow spicular tracts and individual spicules, and is perforated by widely-spaced pores. The dermis is supported over extensive subdermal spaces by spicular tracts that approach the surface diagonally and extend some distance under it. The internal skeleton is composed of many spicular tracts and individual spicules scattered in confusion. The spicules are oxeas 210–380 x 3–9 μ.

Colonies of this species might be confused with colonies of *Hymeniacidon heliophila* of a similar appearance and coloration. The difference in spiculation is distinctive.

This species is attributed to Burton (1930a, p. 489), who discussed the taxonomy of this species. It had often been referred to *Halichondria coalita* Johnston in sponge literature. Hartman (1958) discusses collections from New England and Europe. This species was also described from Beaufort by de Laubenfels (1947, p. 36) as *Calyx poa*, and subsequently was made the type of a new genus *Calyxabra* (de Laubenfels 1949a, p. 49). A study of the type specimen of *C. poa* (U.S.N.M. 22724) shows that *poa* falls to the synonomy of *Halichondria bowerbanki*. This species is distributed from North Carolina northward.

Halichondria melanadocia de Laubenfels

One specimen from off Ocracoke Island (U.S.N.M. 23637), and one fragment from New River fishing banks.

This species is represented by a thin encrustation less than 3 mm thick that covers an area 2 cm square on an oyster shell that was attached to rock. It bears much resemblance to keratose sponges in possessing a conulose surface and a fleshy consistency. It is brown when fresh and in alcohol. The fleshy surface membrane, up to 40 μ thick, bears tangentially placed spicules. It is raised by projecting fibers into large, obvious conules. Extensive subdermal spaces lie between the conules. Large windows in this membrane, up to 2 mm across, probably represent oscules. The pores are microscopic. The interior is fleshy and contains many spaces and a network of multispicular tracts and scattered spicules. Where these tracts extend to the surface, they push the surface membrane out into conules,

and extend beyond their apex as a fiber 60 to 80 μ wide. The spicules are oxeas 180–320 x 3–11 μ.

This species was first described from Tortugas by de Laubenfels (1936b, p. 133). It is primarily West Indian in distribution.

Ciocalypta penicillus Bowerbank

Ten specimens from the Hatteras area, including U.S.N.M. 23624, and one from Beaufort.

This species forms tapering finger-like processes marked by vertical rows of small conules (Fig. 49). Usually a number of such processes are contained in one sponge colony. The figured specimen is 40 mm in height and about 29 mm in diameter. In other specimens, tapering processes are up to 10 cm tall, and by the fusion of several units, the bases measure up to 3.5 cm wide. In life, this species is light gray in color and toughly spongy.

The surface is raised into vertical ridges 1.5 to 2 mm apart which are further produced in low conules less than 1 mm apart. The dermis averages about 75 μ in thickness, but varies according to the quantity of tangentially-placed spicules included. In places, narrow spicular tracts can be seen crossing a transparent membrane, and in others, the dermis is simply a crust of tangential spicules. Large and extensive subdermal spaces are present, through which spicular columns extend to the surface, supporting the dermis. A few pores 200 to 250 μ in diameter occur in the membrane; oscules are apparently represented by gaps in the dermis between ridges. Internally, the skeleton contains a loose arrangement of branching and anastomosing vertical fibers 50 to 100 μ in diameter with many scattered spicules. Stout horizontal fibers 150 to 400 μ in diameter project laterally up to 4 mm beyond the longitudinal skeleton, providing support for the conules. Where the dermis is lost in beach-worn specimens, these projecting fibers give the sponge a peculiar, fuzzy appearance. These fibers are literally filled with spicules. The spicules are styles of two size classes: long styles 375–500 x 5–12 μ, and short styles 260–290 x 3–7 μ.

This species was described by Bowerbank (1862, p. 1105). It is widely distributed in the eastern hemisphere.

Ciocalapata gibbsi, new species

One specimen (type: U.S.N.M. 23666) from New River fishing banks.

The type specimen (Fig. 47) is composed of a series of fingerlike processes whose bases are fused into an encrusting mass about 3 cm in diameter. The whole colony is about 3 cm high. In alcohol, the sponge is colorless and stiff, although the tips of several processes are bent as if they were soft at the time of collection. At the surface is a smooth dermis about 40 μ thick with no evident oscules or pores. The dermis is supported by columns over an extensive subdermal space approximately 600 μ high. These columns are stout, 160 to 260 μ in diameter, and contain many spicules. Where they reach the surface, narrow tracts of spicules radiate in all directions, supporting the dermal membrane. The interior skeleton is a confused mass of spicules. The spicules (Fig. 35) are (1) fusiform styles 360–660 x 10–14 μ, (2) equally abundant long oxeas 360–630 x 8–12 μ, and (3) short oxeas 140–290 x 3–7 μ.

This species is named in honor of Mr. Robert F. Gibbs, Superintendent of the Cape Hatteras National Seashore, National Park Service.

The occurrence of both oxeas and styles in its spiculation places *gibbsi* in the genus *Ciocalapata* as defined by de Laubenfels (1936b). It differs from *C. sacciformis* (Thiele), from the Indo-Pacific region, by possessing large styles rather than small ones, and two classes of oxeas rather than one.

Hymeniacidon heliophila (Parker) de Laubenfels

Many specimens from off Ocracoke Island, and many from Beaufort Harbor, including U.S.N.M. 23627.

This species produces encrusting masses from which finger-like processes 20 to 40 mm high may develop (Fig. 50). Theses processes may fuse laterally and contribute to enlarging the main body of the sponge. This species is pale yellow-orange or bright orange-red. The tissue is soft and easily torn, or fleshy in consistency. The surface is minutely conulose, and hispid with projecting spicules. Oscules 1 to 5 mm in diameter are located at the ends of long canals that are often situated at the tips of processes. The dermal membrane contains many spicules, some placed tangentially and others projecting radially with their points directed outward. The superficial layer may contain many tufts or brush-like masses of spicules and protoplasm that contribute to the conulose appearance. Systems of large subdermal channels up to 2 mm in diameter can often be seen through the dermis. In the interior, spicules are arranged in

tracts, or scattered without order. The spicules are styles 130–320 x 3–8 μ, slightly curved or straight.

Specimens from off Ocracoke Island have a more fleshy consistency and do not exhibit the erect processes that typically bear oscules in Beaufort specimens. Probably both these features are due to environmental influences, for the Ocracoke specimens are exposed to the circulation and wave action of the open ocean, while the Beaufort colonies grow in more protected locations.

Some specimens of this species may be confused with *Halichondria bowerbanki* in the field. However, spiculation quickly distinguishes the two, for the latter species possesses oxeas rather than styles.

This species was first described as *Stylotella heliophila* by Parker (1910, p. 766). George and Wilson (1919) and Miner (1950) have treated it as *Stylotella heliophila* Wilson. It was transferred to the genus *Hymenacidon* by de Laubenfels (1947, p. 34). The record of this species from New River fishing banks (Pearse and Williams 1951) should be transferred to *Oxeostilon burtoni*. We have examined the specimen (U.S.N.M. 23288) recorded as *Hymeniacidon heliophila* for northwest Greenland (de Laubenfels 1942) and found that it is not *H. heliophila*. We have also examined a specimen from Virginia (U.S.N.M. 23341) that was identified by de Laubenfels as this species; it is not *Hymeniacidon heliophila*. This species was included in the paper on Woods Hole sponges (de Laubenfels 1949a) on the surmise that it would be found there, not on the basis of an actual collection.

This species is distributed from North Carolina southward.

Oxeostilon burtoni de Laubenfels

Three specimens from New River fishing banks, including U.S.N.M. 23667.

This species forms very slender, tapering processes arising from an encrusting base (Fig. 51). The figured specimen is 6 cm high. It is orange in life and toughly spongy. The surface is raised into low vertical ridges about 1.5 mm apart. No pores or oscules are evident; the tips of the processes are closed. Microscopically, the surface is hispid with projecting individual spicules. The dermis, 35 to 50 μ in thickness, contains many tangentially placed spicules, with vague tracts running vertically under the crests of the ridges. In the valleys, the dermis is perforated by many evenly distributed pores 90 to 100 μ in diameter. This dermis is supported over extensive subdermal spaces 0.6 to 2 mm high by stout multispicular columns 300 to 400 μ in diameter. The interior region is filled with a fleshy mass of spicules packed without order. The spicules are oxeas 300–500 x 6–10 μ, and styles 300–400 x 5–9 μ, both with points that taper in a series of step-wise stages.

This species was first described by de Laubenfels (1934, p. 15) from off Puerto Rico. In form, *Oxeostilon burtoni* resembles *Ciocalypta penicillus*, but the orange color and spiculation of both styles and oxeas clearly distinguish *O. burtoni* from *C. penicillus*, which is light gray and possesses only styles. *Oxeostilon burtoni* is distributed from North Carolina southward.

ORDER HADROMERINA

The sponges in this order usually contain tylostyles, spicules with an enlarged ball at one end and a point at the other. They may have microspicules in which many rays or spines diverge from an axis or point. The sponge tissues typically have a radiate organization.

Spheciospongia vesparia (Lamarck) Marshall

Several specimens have been collected from the fishing banks off Beaufort Inlet, represented by U.S.N.M. 9016.

This species forms massive cake-shaped or hassock-shaped colonies over 1 meter in diameter. The largest sponges known to science are specimens of this species. The surface is dark brown in color and the interior is a light drab, sometimes with a yellow tint. The tissue has the consistency of cork or wood. The surface is smooth. The upper surface is marked by a series of large excurrent apertures up to 8 or 9 mm in diameter usually grouped in a central depression. The lateral surfaces are marked by clusters of pores 1 to 2 mm in diameter. Each cluster leads through a sieve into a large canal, 1 cm or more in diameter, that in turn leads to many branches in the sponge interior. There is a dense dermis 2 to 5 mm thick in which the spicules are radially arranged. Internally, spicules are found in spicular tracts and packed in confusion.

The spicules are tylostyles 445–600 x 9–10 μ and spirasters 12–15 x 2–4 μ with one to four bends in their shaft. Spirasters are found only lining canals and on the surface, and may be quite rare.

This species was first described as *Alcyonium*

vesparium by Lamarck (1815, p. 78), and transferred to the genus *Spheciospongia* by Marshall (1892, p. 32). It was reported from the Beaufort area by George and Wilson (1919) as *Spirastrella andrewsii*, and as *Poterion atlantica*. This species has been thoroughly redescribed and compared with similar forms by de Laubenfels (1936b, 1950a). We have examined representatives of George and Wilson's species in the National Museum (e.g. U.S.N.M. 9016, 9020, and others), and have confirmed de Laubenfels' identification as *Spheciospongia vesparia*. Miner (1950) treats this species as *Spirastrella andrewsii*.

This species is distributed from North Carolina southward.

Spirastrella coccinea (Duchassaing & Michelotti) de Laubenfels

Two specimens from off Ocracoke Island.

This species is represented by extremely thin colonies encrusting shell and rock dredged from 10 meters depth. The encrustation is less than 1 mm in thickness and slightly less than 4 cm² in area. They were colored red in life, and are dark red-brown in alcohol. The surface is optically smooth with only microscopic openings. It contains many microspicules. The interior is microcavernous with confused masses of tylostyles between the spaces. The spicules are tylostyles 250–300 x 6–8 μ with elongate heads, and abnormal, rod-shaped spirasters about 10 μ long with reduced spines, common in the dermis.

This species was first described as *Thalysias coccinea* by Duchassaing and Michelotti (1864, p. 84), and was transferred to the genus *Spirastrella* by de Laubenfels (1936b, p. 143). Typically this species possesses spirasters with well developed spines. However, Topsent (1932b, p. 558) has described and pictured similar, abnormal spirasters in a specimen of *Cliona vastifica* from the coast of Tunis. They do not warrant taxonomic recognition.

Spirastrella coccinea is widely distributed in the West Indian region.

Anthosigmella varians (Duchassaing & Michelotti) de Laubenfels

Several specimens from off Ocracoke Island.

This species forms thin encrustations on shells and rock. The sponge is usually about 350 μ thick, little more than the length of its spicules. These colonies are less than 1 cm² in area. In its thinnest parts, this species appeared pale orange, almost colorless; but with death, it became a dark red-brown. The surface is hispid with many erect spicules, made more obvious in dead sponges by their contrast with the darkly colored flesh. The extreme thinness obscures other anatomical features. There are no evident oscules or pores. The spicules are tylostyles 210–340 x 6–7 μ in which the heads are pronounced (8 or 9 μ across) and usually subterminal.

This species was first described as *Thalysias varians* by Duchassaing and Michelotti (1864, p. 86), and was transferred to the genus *Anthosigmella* by de Laubenfels (1936b, p. 143). Ordinarily this species possesses spirasters about 24 μ long, but de Laubenfels (1953) has shown that their abundance in this species is variable, and that specimens totally lacking spirasters occur and have often been named as a separate species.

Anthosigmella varians is distributed from North Carolina southward.

Suberites ficus (Johnston) Schmidt

Fig sponge

Two specimens, including U.S.N.M. 23617, dredged from 60 meters depth near the North Carolina-Virginia border, in the Cape Hatteras region.

This species forms flat lobate colonies that grow on sandy bottoms often attached to bivalve shells (Fig. 46). The figured specimen is 13 cm by 5 cm with a maximum thickness of 2 cm near the top. It is straw-yellow in color and firmly spongy in consistency. The surface is smooth and relatively featureless. Several closed oscules can be found that probably measured 2 mm in greatest diameter. The pores are microscopic. The dermis contains many closely packed microspicules. The interior is very compact, containing tracts and vertically arranged columns of macrospicules. The spicules are tylostyles 160–360 x 4–9 μ and centrotylote acanthostrongyles 14–40 x 2–4 μ.

Hartman (1958) discusses the taxonomy of this species. It was first described adequately by Johnston (1842) as *Halichondria virgultosa* (p. 137), *H. suberea* (p. 139), and *H. ficus* (p. 144). The first has priority, but because *ficus* is so well established, Hartman has suggested that the International Commission on Zoological Nomenclature be asked to conserve it. In the meantime, we have followed his example in retaining the commonly used name. This species was transferred to the genus *Suberites* by Schmidt

(1870, p. 76). De Laubenfels (1949a) treated this species as *Choanites ficus*.

This species is distributed from North Carolina northward.

Pseudosuberites sulphureus (Bowerbank) Topsent

Eight specimens from off Ocracoke Island.

This species is represented by encrustations that are about 500 microns thick and as much as 1 cm² in area. They were attached to shells 2 to 3 meters deep. The sponge is fleshy in consistency. In life it is pale orange, and in alcohol it is orangish brown.

The surface is moderately roughened and only slightly hispid. This hispid effect is due to scattered individual spicules projecting slightly beyond the surface. There is a distinct dermal layer about 60 μ thick that contains spicules, most of which are tangentially arranged. A limited number of spicules stand with their tips outward. No pores or oscules are visible, but microscopic pores 40 to 70 μ in diameter were found in sections. The dermis is supported by spicular columns which spread out under the dermal layer. Extensive subdermal cavities occupy a zone 80 to 100 μ thick. The interior contains confused spicule masses, chiefly in the walls of cavities and channels. The spicules are tylostyles 140–320 x 4–5 μ with broad heads.

This species was first described as *Hymeniacidon sulphurea* by Bowerbank (1866, p. 208), and was transferred to the genus *Pseudosuberites* by Topsent (1896, p. 125). In view of its distribution in the eastern Atlantic from Iceland to the Mediterranean, this species appears to be of northern affinities.

Prosuberites microsclerus de Laubenfels

Many specimens from Hatteras Harbor, including U.S.N.M. 23645, and several others from Beaufort.

This species forms thin encrustations on a variety of substrates including rocks, shells, and algae. Most specimens measure about 1 mm in thickness, but by lateral growth they may attain 13 cm in length. The surface is hispid with many erect, projecting spicules. This species is marked by a distinctive chrome-yellow color, which fades somewhat in dried specimens. Dendritic patterns of subdermal canals show on the surface of many larger colonies. However, the oscules must be contractile, for they are not visible. Pores are microscopic. The spicules (Fig. 31) are tylostyles 125–360 x 3–8 μ, with very large heads.

This species was described by de Laubenfels (1936b, p. 143) from Tortugas. However, in 1950 that author placed *microsclerus* in synonymy with *Terpios fugax*. Our rediscovery of this species has permitted us to supplement the original description and to affirm its distinctness. Many features, among them color, surface hispidation, and spicule size serve to distinguish this species from *T. fugax*.

Prosuberites microsclerus is distributed from North Carolina southward.

Terpios fugax Duchassaing & Michelotti

Three specimens from off Ocracoke Island, including U.S.N.M. 23646.

This species forms smooth encrusting or massive colonies that attach to hard substrates. These specimens were attached to a conglomerate rock dredged at 10 meters. The largest formed a very thin encrustation that covered an area equal to 2 cm². When first collected, this sponge presented a shiny black appearance like that of shoe-polish or heavy grease. This color quickly faded through a deep blue, to a beautiful blue-green color in alcohol. It is fleshy in consistency. The surface is optically smooth without detectable openings. Microscopically, individual spicules project well beyond the surface, producing a long hispidation. The internal skeleton is composed of spicules clustered in confused masses. The spicules are tylostyles 160–360 x 2–6 μ. The heads of these tylostyles are typically enlarged and often double or lobate. Three common forms of tylostyle heads are outlined in Figure 33.

The color changes shown by these specimens suggest that color differences which have been utilized as specific distinctions within this genus (see review by de Laubenfels 1950a) may only be transitory states in the death of this species.

This species was first described by Duchassaing and Michelotti (1864, p. 102). De Laubenfels (1950a) placed *Prosuberites microsclerus* in synonymy with this species; however, our study of both forms indicates that they are valid species. The bright chrome-yellow color of *P. microsclerus* is a useful criterion for field recognition. In *P. microsclerus* the spicules are erect upon the substratum, while in *Terpios fugax* many may be tangentially placed, and the colony may become massive.

This species is distributed from North Carolina southward.

Polymastia robusta Bowerbank

One specimen from off Ocracoke Inlet.

This species is represented by a small, convex

encrustation less than 2 cm in diameter, that was attached to conglomerate rock dredged from 10 meters depth. Its color is brownish gray in preservative. The surface is conulose and obviously hispid. Tufts of spicules project about 450 μ beyond the apices of the conules. The conules are spaced about 1 mm apart from one another. No oscules or pores are visible. The spicules are subtylostyles and styles of two size ranges: 200–230 x 2 μ, and 320–864 x 5–12 μ, most being near 500 μ in length.

This species was first described by Bowerbank (1862, p. 822) from the British Isles. It has been reported as extending to North Carolina by Pratt (1935). Typical colonies of this species develop a series of erect, finger-like fistules, but this North Carolina specimen was apparently still too small to produce such processes. Lambe (1896) also recorded only two size classes of spicules in this species, but Proctor (1933) has reported a still larger group of spicules 1080–1360 x 16–24 μ, as well as the two recorded for this juvenile North Carolina specimen.

This species is distributed from North Carolina northward.

Cliona caribboea Carter

One specimen from off Diamond Shoals, one from off Ocracoke Island, and two from New River fishing banks, including U.S.N.M. 23638.

This species is represented by large masses of sponge tissue that do not show the boring habit characteristic of the genus *Cliona*. Certain species of this genus may pass through three stages of development: typical boring colonies that perforate calcareous materials such as coral, shells, or limestone rock, termed the *alpha* stage; colonies that encrust the object they have eroded, termed the *beta* stage; and apparently free-living colonies that have destroyed the object they excavated and have left no trace of their boring habit, termed the *gamma* stage. These specimens fit the last stage in this sequence. The largest specimen was 25 cm by 15 cm and 3 cm thick; Figure 60 shows part of it with the internal structure exposed. It was firm but compressible (cartilaginous) in consistency and was a bright orange-yellow in life. With preservation, its color has changed to dark brown.

The undulating surface is superficially smooth and exhibits many prominent circular areas about 5 mm in diameter. These probably represent low papillae that have contracted. Oscules are visible in the center of some. Pores are microscopic. In the space between these circular areas, fragments of calcareous material may be common. Under the round areas is a tough cortical layer 0.6 to 1.2 mm thick containing spicules packed in confusion. In much of the surface, spicules are closely packed with their points out. In the interior of the sponge are confused masses of spicules and vague tracts in vertical arrangement. Through this internal region large canals up to 6 mm in diameter run vertically and terminate below the circular areas of the surface. Their walls resemble the outer surface in smoothness. The spicules are tylostyles 240–440 x 6–14 μ.

When boring, this species excavates very wide galleries 5 to 6 mm in diameter which the sponge does not completely occupy. A canal remains with a central lumen 2 to 3 mm in diameter.

This species was first described by Carter (1882, p. 346). It is typical of the West Indian region. De Laubenfels identified a specimen of this species from New River fishing banks as *Pseudosuberites melanos* (Pearse & Williams 1951). While that species has a similar form, it has smaller spicules (125–285 x 3–6 μ) and a very different color (slate black). In view of the remarkable color change with death in *Cliona caribboea*, it seems likely that de Laubenfels was misled by the dark brown color of the preserved specimen.

Cliona celata Grant

Many specimens from Hatteras and Beaufort Harbors, including U.S.N.M. 23648.

Like *C. caribboea*, this species can be found boring into calcareous shells, in an encrusting state, or in a massive, free-living form showing little trace of its boring nature. Perforations produced in shells by this species are obvious, measuring 1.5 to 3.0 mm for oscular openings and 0.8 to 1.1 mm for pore openings. Under the surface, connecting galleries may be so extensive that only calcareous pillars connect the outer shell layers. In life, yellow or greenish papillae extend from the perforations, the larger papillae bearing oscules 1 to 1.5 mm in diameter and the smaller ones bearing a cluster of microscopic pores. The galleries contain confused masses of spicules—tylostyles 200–400 x 4–10 μ. Although spirasters have been recorded from European specimens (see Hartman 1958), they have not been observed in North Carolina material.

Most commonly, this species is represented by its boring form, but we have also found sheets

of this species lining the interior of dead barnacle shells and small oyster "boxes". In its massive, freeliving stage, this species possesses a smooth surface marked by many small papillae 2 mm apart. The yellow or yellow-orange color turns dark brown with death.

This species is the most common species of *Cliona* in North Carolina. It can be recognized from the other *Cliona* species by the size of its shell perforations and by its spiculation. All the other species but *C. caribboea* possess spirasters or spirasters and oxeas in addition to tylostyles.

This species was first described by Grant (1826, p. 79). Being a cosmopolitan species, it is distributed both north and south of North Carolina.

Cliona lobata Hancock

Several specimens from Pamlico Sound and Newport River, represented by U.S.N.M. 23639.

This boring sponge is represented only by colonies that have penetrated oyster shells. At the surface, its perforations in the shell fit into two obvious size classes: the larger perforations, 400–600 μ in diameter, are excurrent openings, while the smaller perforations, 150–350 μ in diameter, are incurrent openings. These perforations may be arranged in a dendritic pattern or in a closely-spaced group. The sponge occupying these perforations was pale yellow in color. Its spicules are slender tylostyles 170–210 x 1.5–4 μ, and two sizes of spirasters: those 25–50 x 4 μ with prominent spines spirally arranged, and those 8–25 x 2–3 μ with evenly distributed inconspicuous spines. Both types of spirasters are typically multiply bent or zig-zagged. The larger spirasters and the absence of oxeas distinguish this species from most other North Carolina boring sponges.

Cliona lobata was first described by Hancock (1849, p. 341). It has been collected both north and south of North Carolina.

Cliona robusta Old

Old (1941, p. 9) described this species from three specimens collected in Beaufort Harbor. Only one specimen has been reported since the original discovery, that from South Carolina (Hopkins 1946).

The shell perforations made by this species are small (presumably 0.5 to 1.5 mm in diameter). The live sponge tissue is yellow to orange. The spicules are (1) slender tylostyles 160–200 x 2–4 μ; (2) stout, finely microspined centrotylote oxeas 40–130 x 4–12 μ (the spination or central swelling may be missing); and (3) small, microspined spirasters 8–16 x 1–2 μ. The unusual width of the oxeas serves to distinguish this species of *Cliona*.

Cliona spirilla Old

One specimen (U.S.N.M. 23644) from off Portsmouth Island, and several from Beaufort Harbor.

This boring sponge makes perforations in shells 0.5 to 1.5 mm in diameter. The sponge tissue is light yellow in color. The spicules are (1) relatively rare slender tylostyles 160–260 x 2–5 μ; (2) very abundant large, microspined oxeas 90–200 x 3–8 μ, often somewhat curved and occasionally with a central swelling; and (3) relatively rare microspined or smooth twisted spirasters 9–22 x 1–2 μ.

This species was first described by Old (1941, p. 10) from the Beaufort area.

Because he had difficulty in separating this species from *Cliona vastifica*, Hopkins (1956) treated specimens that fitted this description as

TABLE 2

Spicule dimensions for North Carolina specimens of Cliona *species. (All dimensions in microns.)*

Species	Tylostyles	Oxeas	Spirasters
Cliona caribboea	240–440 x 6–14	—	—
Cliona celata	200–400 x 4–10	—	—
Cliona lobata	170–210 x 1.5–4	—	8–25 x 2–3, 25–50 x 4
Cliona robusta	160–200 x 2–4	40–130 x 4–12	8–16 x 1–2
Cliona spirilla	160–260 x 2–5	90–200 x 3–8	9–22 x 1–2
Cliona truitti	170–230 x 2.5–4	75–130 x 2–5	7–14 x 0.5–2
Cliona vastifica	260–330 x 3–6	50–130 x 2.5–4	6–15 x 1–3
Cliona viridis	200–460 x 3–10	—	10–18 x 0.5–2

the latter species. Wells (1959) experienced a similar difficulty in distinguishing these forms. The possibility arises that *C. spirilla* is only an ecophenotype of *C. vastifica*, but it seems more likely that they are distinct species. Wells found different distribution patterns for the two forms in the Beaufort area, which overlapped in higher salinities. In the Cape Hatteras area, both forms have been collected from shells attached to hard substrates in the open ocean. Their collection from areas of similar salinity casts doubt on any interpretation that one is only a brackish-water form of the other species. The large oxeas of *C. spirilla* almost equal the tylostyles in length and often exceed them in width, while in *C. vastifica* the tylostyles are much longer than the oxeas. Spicule dimensions are compared in Table 2 for these and other *Cliona* species.

Cliona spirilla has been collected in North and South Carolina.

Cliona truitti Old

Several specimens from Beaufort Harbor, and many specimens from Hatteras Harbor, including U.S.N.M. 23640.

This boring sponge produces perforations in shell about 0.5 mm in diameter. The sponge tissue within the galleries may be colored yellow to orange-red. Gemmules can frequently be found in this species. The spicules are (1) slender, often curved tylostyles 170–230 x 2.5–4 μ; (2) slender, smooth or microspined oxeas 75–130 x 2–5 μ, occasionally with a central swelling; and (3) nearly straight microspined spirasters 7–14 x 0.5–2 μ.

Cliona truitti was first described from Maryland waters by Old (1941, p. 10), and was first recorded from North Carolina waters by Wells (1959). It is distributed both north and south of North Carolina.

Cliona vastifica Hancock

One specimen (U.S.N.M. 23643) from off Hatteras Island, many specimens from Pamlico Sound, and several from Beaufort Harbor.

This species produces perforations 0.5 to 1.5 mm in diameter in the shells of oysters and scallops. The perforations may form dendritic patterns on shell surfaces, or they may be distributed apparently at random. The sponge tissue is yellow or orange in life. Often gemmules are found in its tissues. The spicules are (1) abundant slender tylostyles 260–330 x 3–6 μ; (2) slender microspined or smooth oxeas 50–130 x 2.5–4 μ; and (3) zig-zag microspined spirasters 6–15 x 1–3 μ, in which the spines are more prominent at the angles.

This species was first described by Hancock (1849, p. 343). Hopkins (1956) included specimens of *C. spirilla* in his treatment of the distribution of this species. Although the spiculation of these two species is similar, they may be separated on the basis of relative sizes of the spicules (Table 2). *Cliona vastifica* is distributed both north and south of North Carolina.

Cliona viridis (Schmidt) Gray

One specimen (U.S.N.M. 23614) from Beaufort Harbor.

Like *Cliona caribboea* and *C. celata*, this species may form upright massive sponges as well as colonies that bore into shell. This North Carolina specimen combines the features of both growth forms. Its base occupies galleries in an oyster shell while a large mass of ascending lobes 7.5 cm by 9 cm rises to a height of 6 cm above the shell. Perforations in the oyster shell measure 0.4 to 1.6 mm in diameter and occur in a dendritic pattern at the edge of the shell. The basal portion of the sponge mass incorporates a few pieces of shell. The erect part of the body is divided into many firm narrow upright lobes that fuse freely. The lobes end in soft hollow papillae 2 to 8 mm in diameter and up to 15 mm high. The upper edges of these papillae are somewhat thickened. This specimen is light gray in alcohol, and of a fairly firm consistency. The color of this species in life is typically yellow, green, or brownish.

The surface is minutely roughened and contains many projecting spicules. The dermis is perforated by oscules about 1 mm in diameter on the upper margin of the lobes. The interior contains tracts of spicules with many canals 100 to 300 μ in diameter. The spicules are tylostyles 200–460 x 6–10 μ, with heads quite swollen and exhibiting a variety of forms, and small spirasters about 10–18 x 0.5–2 μ, found only in the basal part of the sponge. Spirasters are particularly abundant in papillae that project through the shell perforations.

This species was first described as *Vioa viridis* by Schmidt (1862, p. 77), and was transferred to the genus *Cliona* by Gray (1867, p. 525). This specimen was described by George and Wilson (1919, p. 140) as *Suberites undulatus*, and Miner (1950) treats it under that name. De Laubenfels (1947) suggested that the forms described by

George and Wilson were unusual specimens of *Microciona prolifera*. The scarcity of spirasters in erect portions of this sponge makes its correct identification difficult.

In summary, the spiculation of the species of *Cliona* collected in North Carolina is compared in Table 2.

ORDER EPIPOLASIDA

Sponges of this order possess a radiate architecture with a well-developed fibrous cortex. They contain oxeas as principal spicules and never have tylostyles. North Carolina representatives possess asters of various kinds as microspicules. They lack the four-rayed spicules characteristic of the next order, Choristida.

Dorypleres carolinensis, new species

One specimen from New River fishing banks (type: U.S.N.M. 23668).

The type specimen (Fig. 58) is a cushion-shaped mass 5 cm in diameter and 3 cm thick that was attached to a rock. It is firm in consistency and pink-colored in alcohol. While the upper surface is smooth, the lateral surfaces are somewhat coarsely roughened. Microscopically, the entire surface is hispid with projecting spicules. Oscules 1 to 2 mm in diameter are visible in lateral regions, lying between slightly elevated ridges. Pores are microscopic. The stout cartilaginous cortex contains a pallisade of erect spicules. The interior region is a pulpy mass of spicules arranged in radiating bundles. The spicules (Fig. 34) are (1) oxeas 380–1500 x 7–20 μ, (2) euasters about 20 μ in diameter, having six to eight tapering rays with blunt ends (sometimes with lumpy rays), and (3) characteristic delicate euasters 30–70 μ in diameter with five to eight very thin rays that bear minute recurved hooks near their ends.

This species fits into the genus *Dorypleres* as defined by de Laubenfels (1936b) on the basis of its possession of two classes of astrose microspicules. It differs from *D. serpentina* (Wilson) from the Philippines in not possessing the irregular, sinuous spicules for which that species was named, and in other details of spiculation and form. *Dorypleres splendens* de Laubenfels from the western Pacific possesses smaller oxeas (150–610 x 4–10 μ), smaller asters (7–8 μ, 15 μ, and 18 μ), and no spicules resembling the unusual asters characteristic of *D. carolinensis*.

Trachygellius cinachyra de Laubenfels

Three specimens from New River fishing banks, including U.S.N.M. 23669.

This species forms subspherical colonies marked by prominent apertures (Fig. 53). The figured specimen is 6 cm in diameter; the largest specimen, 9 cm in diameter. This species is colored ochre-yellow in life, and dull drab in alcohol. It has a firm, cartilaginous consistency. There is a very thin dermis. On the lateral surfaces there are prominent openings 1 to 3.5 mm in diameter, often with a thickened spicular collar 2 mm high surrounding the opening. A large contractile opening (2 to 3 mm across in one incompletely closed example) is present in the center of the upper surface, often in a slight depression. The spicules are oxeas 2–4 mm long and 4–40 μ wide, and abundant microspined sigmas 6–11 μ long.

Trachygellius cinachyra was first described by de Laubenfels (1936b, p. 158) from Tortugas. De Laubenfels identified these specimens as *Craniella crania*, that record from New River fishing banks (Pearse & Williams 1951) should be transferred to this species. Judging from the similarity of the photograph published of *Cinachyra cavernosa* in de Laubenfels' paper on Bermuda sponges (1950a, fig. 7, upper left), the authors suspect that *Trachygellius cinachyra* has also been collected at Bermuda and confused with specimens of *Cinachyra cavernosa*. The fact that the present specimens were mistakenly identified as *Craniella crania* makes a similar misidentification credible. Both *Craniella* and *Cinachyra* possess the four-rayed spicules typical of the Choristida, which are absent in the Epipolasida. Only a brief check on spiculation is needed to separate *Trachygellius cinachyra* from them.

ORDER CHORISTIDA

The sponges of this order are marked by the possession of four-rayed "triaene" spicules among the macrospicules. Like the preceding order, they exhibit a radiate architecture with a well-developed fibrous cortex, and possess asters of various kinds as microspicules.

Myriastra fibrosa (Schmidt) de Laubenfels

One specimen from New River fishing banks, U.S.N.M. 23670.

This sponge is a semi-encrusting, flattened mass 5 cm long, attached to an oyster shell. In alcohol, it is pale brown and firm in consistency.

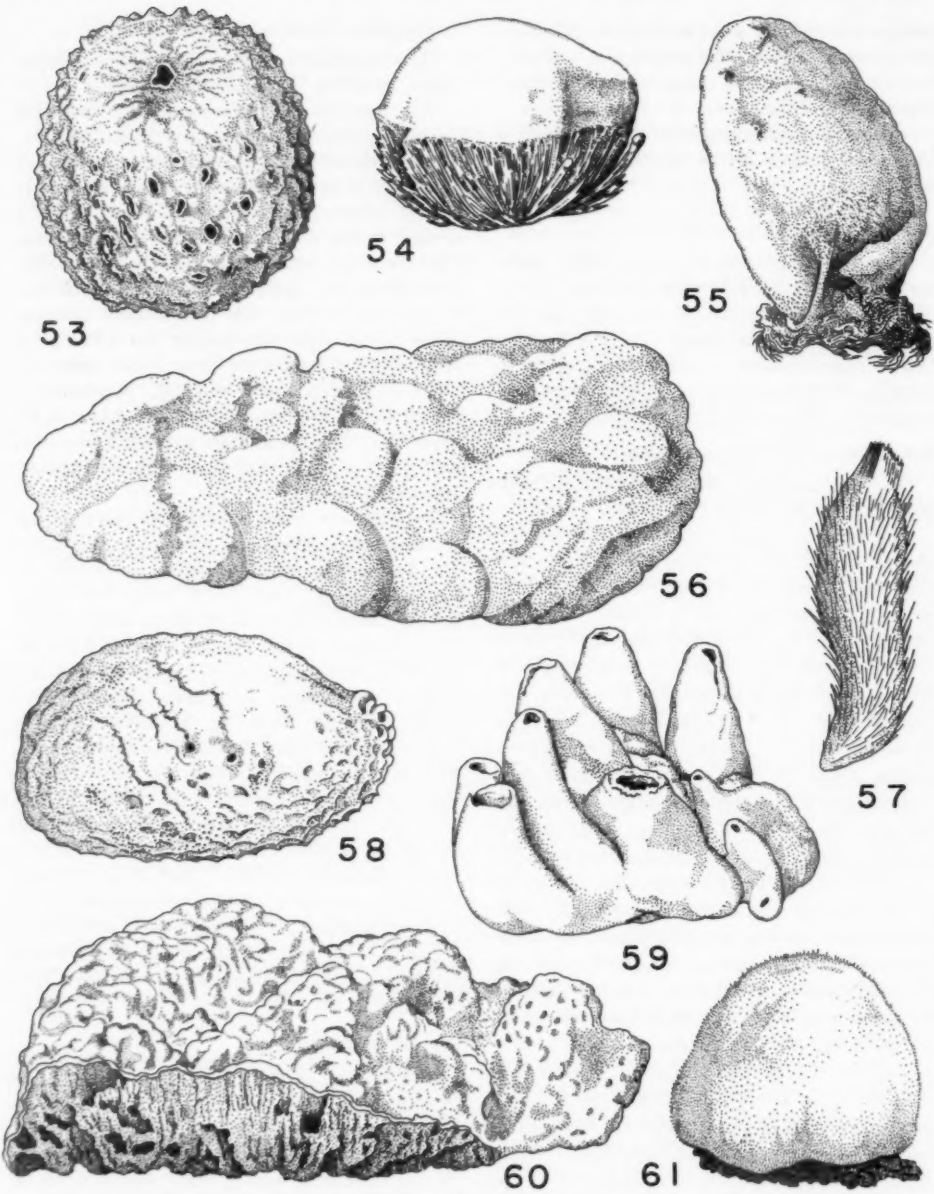

FIG. 53. *Trachygellius cinachyra.* FIG. 54. *Cinachyra cavernosa.* FIG. 55. *Craniella laminaris.* FIG. 56. *Geodia gibberosa.* FIG. 57. *Scypha barbadensis.* FIG. 58. *Dorypleres carolinensis.* FIG. 59. *Leucetta floridana.* FIG. 60. *Cliona caribboea.* FIG. 61. *Craniella crania.*

Many worm tubes, bryozoans, shell fragments, and sand grains are incorporated into the stout cortex which is 700 μ thick. The surface is smoothly porous, or may be punctate and irregular. The two oscules are 2 mm in diameter and ringed by a smooth border. Under the dermal cortex is a layer of subdermal spaces. The spicules are (1) orthotriaenes with one long ray

640–800 x 12–24 μ, and three short rays 20–60 μ long, diverging at right angles; (2) anatriaenes with the long ray 720–1000 x 12–20 μ, and recurved short rays 20–50 μ long; (3) oxeas 700–950 x 12–18 μ, primarily in the interior; (4) slightly curved oxeas 168–230 x 6–8 μ, radially arranged in the cortex; and (5) euasters 10–16 μ in diameter, with 8 to 12 rays, in the cortex.

This species was first described as *Ancorina fibrosa* by Schmidt (1870, p. 67). Wilson (1902, p. 384) treated it as *Pilochrota variabilis*. De Laubenfels (1936b, p. 169) transferred this species to the genus *Myriastra*. It is typical of the West Indian region.

Geodia gibberosa Lamarck

Three specimens from the Hatteras area, and one from New River fishing banks (U.S.N.M. 23671).

This species forms large, massive colonies that resemble rounded calcareous rocks (Fig. 56). The figured specimen measures 23 cm by 16 cm and 5 cm at its greatest thickness. It is chalky white with a tint of greenish yellow. It is stony in consistency, due to the great quantity of spherical spicules in the cortex. The surface is an undulating, superficially smooth crust. Oscules 0.5 to 1 mm in diameter occur in groups that occupy slight depressions up to 10 cm^2 in area. Much smaller pores are evenly distributed over most of the surface. The dermal cortex, about 1 mm thick, is a solid crust composed of many closely packed sterrasters. Extending inward from the cortex are many radially arranged macrospicules. The interior is a confused mass of spicules. The spicules are (1) oxeas 900–1800 x 20–27 μ; (2) orthotriaenes with long ray 1100–1300 x 20–32 μ and short rays 140–200 μ long diverging at right angles; (3) sterrasters 50–65 μ in diameter, typical of the cortical region; and (4) euasters 12–22 μ in diameter, with rays tapering to slender points, characteristic of the interior.

Geodia gibberosa was first described by Lamarck (1815, p. 333). It is a West Indian species.

Cinachyra cavernosa (Lamarck) Topsent.

Two specimens from New River fishing banks, including U.S.N.M. 23672.

This species is represented by two ovoid specimens that were attached to a rock. The figured specimen (Fig. 54) is 25 mm long, 20 mm wide, and 20 mm high. The living sponge is yellow-orange and hard in consistency; in alcohol the color is dull drab and the projecting spicules upon its surface form dark gray patches. The upper surface is hispid with projecting spicules. There is a contractile fibrous cortex. Abundant elliptical openings, 200–300 μ in diameter are found around and slightly above the equatorial zone. A larger opening 3 mm in diameter was found at the more pointed end of one specimen. According to de Laubenfels (1950a), this opening is the incurrent aperture, and the former openings are oscules. The interior of this sponge is radiate, with large, obvious bundles of spicules perpendicular to the surface. The spicules are (1) abundant oxeas 3500–4000 x 35–35 μ; (2) anatriaenes and (3) protriaenes, long rays up to 3000 x 8–20 μ, short rays 50–300 μ long, in the surface plush; (4) many slender growth stages of (1), (2), and (3); and (5) sigmoid spirasters 10 μ long.

This species was first described as *Tethya cavernosa* by Lamarck (1815, p. 70), and was transferred to the genus *Cinachyra* by Topsent (1931, p. 5). It is circumtropical in distribution.

Craniella crania (Müller) Schmidt

Fragments of a specimen from off Beaufort Inlet, and one specimen from Cape Hatteras beach.

This species forms compact, erect ovoid or spheroid colonies up to 5 cm in height that attach to a hard substrate (Fig. 61). The colonies are stony in consistency and light gray-green in color. The surface is strongly hispid and without obvious oscules or pores. There is a thin cortical layer. Internally, bundles of spicules radiate from the center of the sponge, and some scattered spicules occur between them. The spicules are principally oxeas 1000–2000 x 15–20 μ, less abundant protriaenes and anatriaenes of about the same length with short rays about 50 μ long, and abundant sigmoid spirasters 15 μ long.

This species was first described by Müller (1776, p. 255) as *Alcyonium cranium*. Specimens collected from New River fishing banks and identified by de Laubenfels as *Craniella crania* (Pearse & Williams 1951) are actually *Trachygellius cinachyra*. In 1947, de Laubenfels placed *Tetilla laminaris* George & Wilson (1919, p. 142) in synonymy with *Craniella crania*. That sponge is treated below as a distinct species.

Craniella crania is distributed north of North Carolina along the American coast.

Craniella laminaris (George & Wilson) new combination

Several specimens from the Beaufort area represented by U.S.N.M. 23615.

This sponge forms flattened colonies that attach in sand or mud bottoms. The figured specimen (Fig. 55) is a very young specimen only 5 cm high and less than 1 cm thick. Older specimens form a horizontally elongated lamella the size and shape of a man's hand, 2 to 3 cm thick with the upper edge somewhat thickened. It is straw-yellow in color and firm or cartilaginous in consistency.

The surface is superficially smooth, but shows many projecting spicule ends under a lens. Oscules 0.5 to 2 mm in diameter may be found near the upper margin. The thin dermal membrane is also perforated by many microscopic pores. In the interior are vertical spicular tracts, radiating spicular bundles, and scattered individual spicules. Radial tracts extend downward from the base to form rootlets up to 5 cm long, composed of spicules and attached sand grains. The spicules are (1) stout oxeas 600–2300 x 16–20 μ; (2) slender oxeas 800–1000 x 8 μ; (3) protriaenes, long ray 500–2000 x 3–8 μ, short rays 20–50 μ; (4) anatriaenes, long ray 1500–3500 μ, short rays 32 μ; and (5) sigmoid spirasters 12 μ long.

This species was first described by George and Wilson (1919, p. 142) as *Tetilla laminaris*. De Laubenfels (1947) noticed the similarity of its spiculation to that of *Craniella crania* and placed it in synonymy with that species. Spiculation places *laminaris* in the genus *Craniella* as defined by de Laubenfels (1936b), but growth form, color, and habitat differences clearly separate the two forms. *Craniella laminaris* is flattened, orange or yellow colored, and grows in mud or sand, attached by a specialized set of spicule-bearing rootlets; *C. crania*, on the other hand, is ovate or spheroid, light gray-green, and grows attached to a solid or firm substrate.

Craniella laminaris is distributed from North Carolina southward.

CALCIOSPONGIAE

Sponges of this class contain calcareous spicules.

Scypha barbadensis (Schuffner) de Laubenfels

Many specimens from the Cape Hatteras area, including U.S.N.M. 23628, one from Beaufort Harbor, and one from New River fishing banks.

This species attaches to rocks, shells, hydroids, and bryozoan colonies. It forms vase-shaped colonies, wider in the middle than at the ends, with a crown of spicules projecting around the apical oscule (Fig. 57). The figured specimen is 4 mm high; these North Carolina specimens all measure less than 10 mm. They are brownish-white and somewhat stiff in consistency. The surface is microscopically hispid. The apical oscule, about one-half the diameter of the body, is surrounded by a crown of oxeas. The spongiocoel, a cloacal cavity running the length of the body, occupies the central axial region. A system of channels and chambers (50–70 x 80–95 μ) is contained in its walls. The skeleton consists of a colloidal jelly and calcareous spicules. The spicules are (1) regular triaxons with rays 70–90 μ in the side walls, (2) oxeas 260–280 x 5–7 μ projecting from the surface, (3) oxeas at least 280 x 1–2 μ forming the oscular crown, (4) triaxons with lateral rays 60–80 μ and the basal ray longer, 90–130 μ, and (5) tetraxons with lateral rays 40–75 μ, basal ray 120 μ, and a very short, curved ray 30 μ long, that projects into the spongiocoel.

This species was first described as *Sycandra barbadensis* by Schuffner (1877, p. 429), and was transferred to the genus *Scypha* by de Laubenfels (1950a, p. 145). It is distributed from North Carolina southward.

Leucetta floridana (Haeckel) Dendy & Row

Two specimens from New River fishing banks, including U.S.N.M. 23673.

This species is represented by a group of tapering hollow lobes, closely massed, fused at their bases, and measuring 35 mm across (Fig. 59). This species is pinkish-white in life, becoming brownish in alcohol. Its consistency is brittle. Superficially the surface is smooth, but it is microscopically hispid with projecting spicules. Oscules 2 to 5 mm in diameter are located at the tips of the lobes, about 15 mm from the base. The interior is dense and bears many canals. The spicules are chiefly regular triaxons with rays 90 x 9 μ to 600 x 50 μ. In the cloacal lining, alate triaxons occur in which two of the rays are bent. In addition, there occur tetraxons with the fourth ray much shorter than the other rays.

This species was first described by Haeckel (1872, p. 144) as *Leucaltis floridana*, and was transferred to the genus *Leucetta* by Dendy and

Row (1913, p. 734). The occurrence of tetraxons was not recorded by de Laubenfels (1950a) in his description of specimens from Bermuda. *Leucetta floridana* is a West Indian species.

Leucosolenia canariensis (Miklucho-Maclay) Dendy & Row

Several specimens from off Ocracoke Island, including U.S.N.M. 23634.

This species is represented by a very thin encrustation attached to shells collected at 2 to 3 meters depth. It typically forms masses composed of a series of fused tubes. The color in life is usually yellow, but these North Carolina specimens, collected in mid-August, exhibited the orange coloration noted for specimens in mid-August in Bermuda (de Laubenfels 1950a). Probably this coloration is associated with some phase of the reproductive cycle. The surface is microscopically hispid from projecting spicule ends. Oscules and pores are not visible in these specimens. The skeleton is composed of many triaxons with rays 60–75 x 6–7 μ and irregular, curved oxeas 120–150 x 4–6 μ. These specimens differ from the Bermuda forms described by de Laubenfels (1950a) in having no tetraxons, and in possessing numerous curved oxeas.

This species was first described as *Nardoa canariensis* by Miklucho-Maclay (1868, p. 230), and was transferred to the genus *Leucosolenia* by Dendy and Row (1913, p. 796). It is distributed throughout the West Indian region.

II. Distribution

Of the seventy species of sponges collected in North Carolina waters, six are here described as new: *Hemectyon pearsei*, *Axinella bookhouti*, *Pseudaxinella wilsoni*, *Teichaxinella grayi*, *Ciocalapata gibbsi*, and *Dorypleres carolinensis*. In addition, a large percentage of the species treated are newly recorded from North Carolina. Most of these new records extend the known distribution of the species 500 miles or more along the western Atlantic seaboard.

Records of the following species extend ranges southward to North Carolina: *Haliclona loosanoffi* from Maryland; *H. canaliculata*, *Halichondria bowerbanki*, and *Suberites ficus* from Long Island Sound; *Haliclona palmata* from Woods Hole, Massachusetts; and *Eurypon clavata* and *Pseudosuberites sulphureus* from Iceland. Ranges are extended northward to North Carolina for the following species: *Halicona viridis*, *Callyspongia vaginalis*, *Pseudaxinella rosacea*, *Spirastrella coccinea*, and *Anthosigmella varians* from the Bahamas and Florida; *Haliclona areolata*, *Xestospongia halichondrioides*, *Axinella reticulata*, and *Oxeostilon burtoni* from the Puerto Rican area; *Microciona juniperina*, *Endectyon tenax*, *Halichondria melanadocia*, *Terpios fugax*, *Cliona viridis*, *Scypha barbadensis*, and *Leucosolenia canariensis* from Florida and the West Indies; and *Dictyociona adioristica*, *Axinella polycapella*, *Homaxinella rudis*, *H. waltonsmithi*, *Prosuberites microsclerus*, and *Trachygellius cinachyra* from the Gulf Coast of Florida. This is the first record of *Mycale cecilia* from the Atlantic Ocean, the other records being from the Pacific Coast of Panama and from Hawaii. The record of *Ciocalypta penicillus* is the first from the Western Hemisphere, other records of that species being widely distributed in the Eastern Hemisphere, including the eastern Atlantic. *Pleraplysilla minchini* is recorded for the first time in the western Atlantic.

Because of the geographic location of North Carolina, an analysis of geographic affinities of its sponge fauna is appropriate. The most prominent feature of North Carolina's cuspate coastline is Cape Hatteras, long recognized as the boundary between the Virginian and Carolinian biogeographic provinces (Figs. 62 & 63). The faunas of these two marine geographic provinces are undoubtedly sufficiently distinct for separate recognition, but there is some question as to whether they are not sub-divisions of greater geographic areas (Ekman 1953, Stephenson & Stephenson 1954). The Gulf Stream leaves the Atlantic coast in the Cape Hatteras region, flowing in a north-easterly direction; the warmth of the Gulf Stream system is an important factor to the climate of the marine waters of the southeastern states. North of Cape Hatteras cooler coastal water moves southward to North Carolina, then swings eastward to parallel the western edge of the Gulf Stream. North Carolina is the location of a transition between these two current systems, their respective temperature regimes, and provincial (or regional) faunas.

For geographic analysis, each species is assigned to a group of similar distribution, this distribution being adjudged on the basis of other collections and published records. Where records are not accompanied by any description, we cannot be certain of an identification, and our assignment to a distribution type consequently may be in error. The lack of published studies of

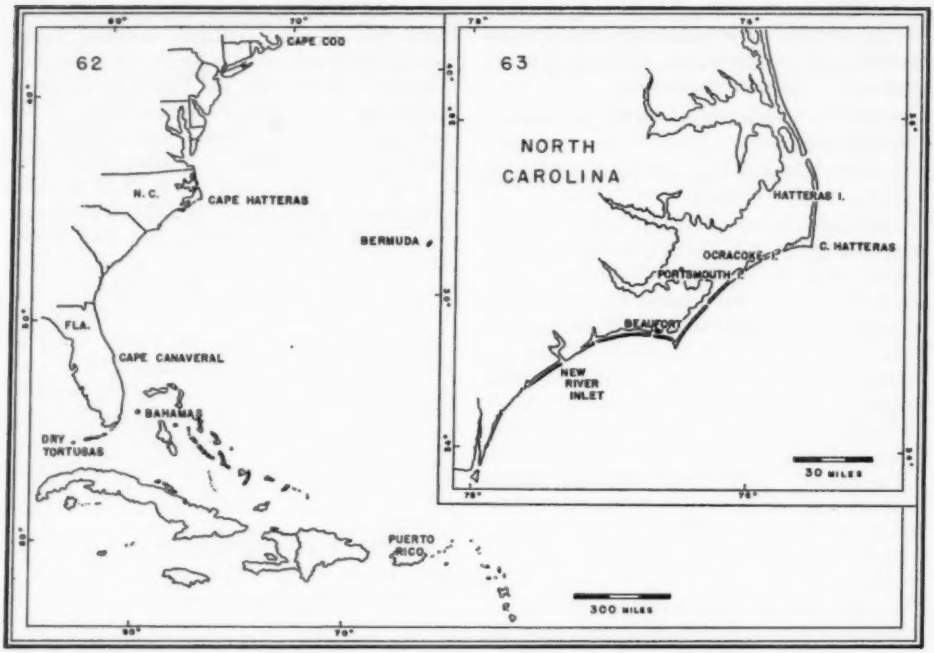

FIG. 62. Chart of western North Atlantic Ocean showing southeastern coast of North American and part of the West Indian region.

FIG. 63. Chart of North Carolina coast showing areas of sponge collections.

sponges between Long Island Sound and North Carolina on the one hand, and between the southern tip of Florida and North Carolina on the other hand, makes such an analysis of distribution along the Atlantic coastline hazardous and difficult. We have assigned each sponge species a distribution type on the basis of available information. Figure 62 shows the relationship of North Carolina to the areas under discussion.

Ten species represented by North Carolina collections have been collected only in the Carolinian province, that area lying between Cape Hatteras and Cape Canaveral, Florida. These species are *Adocia tubifera*, *Hemectyon pearsei*, *Tenaciella obliqua*, *Axinella bookhouti*, *Pseudaxinella wilsoni*, *Teichaxinella grayi*, *Ciocalapata gibbsi*, *Cliona robusta*, *C. spirilla*, and *Dorypleres carolinensis*. It seems likely that further collecting would show that many of these species are truly West Indian forms, and should be included in that group of sponges.

Only two species, *Haliclona canaliculata* and *H. loosanoffi*, have been collected only in the Virginian province, that is between Cape Hatteras and Cape Cod. Eight other species are distributed from North Carolina northward beyond Cape Cod; they are *Haliclona palmata*, *Microciona prolifera*, *Eurypon clavata*, *Halichondria bowerbanki*, *Suberites ficus*, *Pseudosuberites sulphureus*, *Polymastia robusta*, and *Craniella crania*. De Laubenfels (1949a) described the distribution of northern sponges as stopping in the vicinity of New Jersey, and the sponge fauna of North Carolina as being composed of southern species and widespread eurythermal Atlantic Coast forms. In this introduction to a discussion of the algal flora of the New England coast, Taylor (1957) described the distribution of algae along the Middle Atlantic Coast in similar terms, and attributed the scarcity of algae to an absence of suitable substrates. Natural hard substrates are scarce between New York and North Carolina, this part of the coast being dominated by sandy beaches and low marsh-lands. However, many hard substrates have been provided by man as pilings, groins, jetties, navigation aids, and inadvertently an occasional wrecked ship. These structures present a habitat for a number of spon-

ges, and suitable currents and water temperatures have allowed the establishment of northern species on such structures as far south as the North Carolina Capes. A southerly-flowing current along the eastern seaboard north of Cape Hatteras supplies a slow but gradual transport toward this region (Bumpus 1955). Its colder waters provide a region of lower temperatures north of Cape Hatteras that contrasts with conditions south of the Cape, and the effect of summer warming of these inshore surface waters is often counteracted by an upwelling of colder subsurface water (Wells & Gray 1960). The combination of low winter temperatures (as low as 8°C) and moderated summer temperatures on the northeastern coast of North Carolina permits the survival of northern or boreal species of animals. Their collection moves the southern limit for these species from New Jersey south to North Carolina.

In terms of geographic affinities, the largest group of sponges represented in North Carolina collections is composed of forty-one southern species whose range extends southward to the West Indies. They are *Spongia graminea, Phyllospongia lamuga, Ircinia campana, I. fasciculata, Dysidea fragilis, Aplysilla sulfurea, Haliclona areolata, H. viridis, Xestospongia halichondrioides, Callyspongia vaginalis, Rhizochalina oleracea, Xytopsues griseus, Microciona juniperina, Thalyseurypon carteri, Dictyociona adioristica, Endectyon tenax, Axinella polycapella, A. reticulata, Homaxinella rudis, H. waltonsmithi, Pseudaxinella rosacea, Phakellia folium, Higginsia strigilata, Halichondria melanadocia, Hymeniacidon heliophila, Oxeostilon burtoni, Spheciospongia vesparia, Spirastrella coccinea, Anthosigmella varians, Prosuberites microsclerus, Terpios fugax, Cliona caribboea, C. viridis, Trachygellius cinachyra, Myriastra fibrosa, Geodia gibberosa, Cinachyra cavernosa, Craniella laminaris, Scypha barbadensis, Leucetta floridana, and Leucosolenia canariensis.*

Suitable conditions for the northward transport of larvae of these species are provided by the Gulf Stream system, specifically by the Florida Current that roughly parallels the edge of the continental shelf along the southeastern states (Iselin 1936). North of Cape Hatteras this "ocean river" flows northeasterly, and has much less influence upon inshore water and its fauna. The North Carolina Capes extend across much of the continental shelf, so that coastal North Carolina is much closer to the Florida Current than the coastal regions of South Carolina, Georgia, and northern Florida where the continental shelf is much wider. The Florida Current warms these coastal regions and often moderates wintertime low temperatures. By its proximity to the main axis of the Florida Current, coastal North Carolina stands in a position to receive the maximum benefit from this warming effect of its waters. While winter lows may exclude certain southern species from the shallow inshore waters of South Carolina and Georgia, the same species may survive in coastal North Carolina waters, as a result of the slightly higher temperature conditions here. The barnacle *Balanus tintinnabulum* exhibits this type of distribution on the east coast, being found at the southern tip of Florida and at the North Carolina Capes but absent from the intervening coast—apparently as a result of the temperature requirements of that species (Woods Hole Oceanogr. Inst. 1952). Whether any of these forty-one sponge species exhibits a similar distribution is unknown, for no general studies of sponges have been published for the intervening coast. It is evident that the Florida Current produces temperatures high enough in coastal North Carolina for the development of a large number of sponge species typical of the West Indian region.

An additional species probably should be inwith these southern forms; *Mycale cecilia* has been collected only at Hawaii and at the Pacific end of the Panama Canal. It seems likely that further collecting in the Caribbean or in the West Indies will show that this species is represented on both sides of Central America, and that this North Carolina record represents only a northern outpost of a West Indian distribution.

Six species are found both north and south of North Carolina: *Haliclona permollis, Lissodendoryx isodictyalis, Cliona celata, C. lobata, C. truitti,* and *C. vastifica.* They represent eurythermal species distributed from Cape Cod to the West Indies or wide-spread, cosmopolitan species. *Ciocalypta penicillus* has not been collected previously in the Western Hemisphere but is cosmopolitan in the Eastern Hemisphere. Previously, *Pleraplysilla minchini* has only been reported off the coast of France. For geographical analysis, these eight species must be treated together, as neither northern nor southern species. It is of interest that the first three named species in this group are three of the most common species at Beaufort; though numerically small, this group

TABLE 3

Numerical analysis of geographic affinities of sponge species collected in North Carolina

Distribution type	Hatteras	Beaufort	New River fishing banks	North Carolina
Northern	10	5	0	10
Southern	22	23	24	45
Both N & S	7	8	0	8
Endemic to N. C.	0	2	5	7
Total	39	38	29	70

forms an important component of North Craolina's sponge fauna.

In this discussion of geographic affinities along the Atlantic Coast, Bermuda occupies a unique position. It has not been included in the account above, because it lies more than 700 miles from the nearest point on the American mainland, which happens to be Cape Hatteras. However, the sponges of Bermuda have been very adequately studied by de Laubenfels (1950a); and because of its geographic position, they are valuable for comparison with the North Carolina sponge fauna. Nineteen species have been collected in both areas. They are *Ircinia fasciculata, Dysidea fragilis, Aplysilla sulfurea, Haliclona permollis, H. viridis, Callyspongia vaginalis, Lissodendoryx isodictyalis, Xytopsues griseus, Eurypon clavata, Homaxinella rudis, Pseudaxinella rosacea, Hymeniacidon heliophila, Spirastrella coccinea, Terpios fugax, Cliona caribboea, Geodia gibberosa, Cinachyra cavernosa, Leucetta floridana,* and *Leucosolenia canariensis.* As in North Carolina, both species of northern affinities and of southern affinities are present, as well as several distributed widely along the Atlantic Coast. The sponge fauna of Bermuda is transitional.

In Table 3, North Carolina sponge collections are analyzed numerically for geographic affinity. Of a total of seventy species collected in North Carolina, forty-five are southern, i.e. are distributed from North Carolina southward, and ten are northern, i.e. distributed from North Carolina northward. The sponge fauna of North Carolina is predominantly southern in affinity, but with a sizeable representation of northern species and widespread eurythermal species. The transitional nature of its sponge fauna reflects its geographic position.

Further analysis of the species from each of the three areas collected also appears in Table 3. More northern species are represented in collections from the Hatteras area, the most northern collection area; only half that number are represented from Beaufort; and none have been collected at the New River fishing banks, the southernmost collection area. The representation of southern species is least in the Hatteras area, and gradually increases numerically farther south. However, the overall importance of southern species increases at the most southern collecting ground, the only other forms being newly described species that may fall into this southern distribution type with further collecting. As one might expect from their geographic positions, in the Hatteras area the northern influence is strongest, though still numerically dominated by southern species, while the New River fishing banks are almost exclusively southern in affinity, the Beaufort area being intermediate in representation.

The variety of collecting methods used and of habitats examined influences the number and type of sponges collected in any area. In the Hatteras area, rock jetties, shells, pilings, and algae in Hatteras and Ocracoke Harbors have yielded many sponges. In Pamlico Sound, shells, algae, *Zostera* blades, and channel markers have been productive of species tolerant to brackish waters.

Of particular interest have been several wrecks of metal ships located on or just off the outer beach of Rodanthe, Waves, Ocracoke, and Portsmouth. These wrecks are oases in the midst of extensive sand bottoms for organisms that require a firm substrate and that cannot tolerate shifting sand. In the Hatteras area they provide the only easily accessible habitat for sponges in water of ocean salinities. In addition to these wrecks, there are an unknown number of metal wrecks totally submerged that may be partially buried in sand but still provide some attachment for sessile organisms, including sponges. Although collections have not been made from these wrecks, many of the fresh sponges collected on the Hatteras beach undoubtedly come from such a habitat.

Collections have also been made of sponges from conglomerate rock dredged from 10 meters depth off Ocracoke Inlet. Outcroppings of this conglomerate rock similarly provide scarce attachment for sessile organisms in the open ocean. Many sponges collected on the beach in fresh condition bear fragments of these conglomerate masses in their base, or are attached to alcyonar-

ian colonies that bear similar fragments, indicating that they were attached to these outcroppings. Several sponges have been taken in shrimp or flounder trawls by commercial fishing vessels.

In addition, a number of sponges have been recovered from beach drift in the Hatteras area, where many organisms accumulate after severe northeast storms. Although it is possible that some of these had been carried considerable distances, most probably came from nearby habitats. The fact that many contained bits of nearby substrate in their bases, that sponge accumulations usually were restricted to a short length of beach opposite charted hard bottoms, and that many of these sponges were fresh and often began changing color in one or two hours, indicates that they were of local origin. Usually this beach drift included, besides sponges, live "higher invertebrates" which probably came from the same hard substrates and were in equally good condition.

In the Beaufort area, many sponges have been collected on rock jetties, pilings, shells, and algae in Beaufort Harbor and Beaufort Inlet. Sand and shell bottoms in Back and Bogue Sounds and Newport River have yielded sponges to dredging. De Laubenfels (1947) dredged several species from a depth of 5 meters in the Atlantic Ocean off Bogue Bank, and George and Wilson (1919) reported on some sponges dredged from offshore fishing banks. George and Wilson also collected some sponges in beach drift on Bogue Bank near Beaufort Inlet.

Pearse and Williams (1951) collected all sponges reported from the New River area. These were collected from 4 to 17 meters depth by diving with a helmet. Sponges were attached to a reef composed of fossiliferous marl and *Vermicularia* tubes. Apparently little attention was paid to encrusting sponges.

Soft bottoms have been dredged only in the Beaufort area; but because sponges usually attach to hard bottoms, and because only one sand-loving species has been collected there, this collection has only a minor effect upon faunal lists.

The intensive collection of deep water sponges at the New River fishing banks exceeds the coverage of similar offshore reefs farther north, and no shallow water or intertidal sponges are included. Because the fauna of this reef is almost exclusively southern in affinity, the representation of southern forms may be greater in proportion to widespread Atlantic coast species than it would in a collection that involved shallow water or intertidal forms from that area. It appears that deep water sponges on the North Carolina coast are likely to be southern species, and that shallow water on intertidal sponges are likely to include northern and widespread Atlantic coast species as well as southern forms. The proportional representation of these groups will influence to some extent the geographic affinity of a collection. However, this effect is not enough to account for the entire picture of sponge affinities as indicated in Table 3.

Fifty-five species meet their distributional limit in North Carolina. Of the ten northern species collected in the Hatteras area, several are represented by smaller colonies in the Beaufort area, some are found only in the winter, and others have never been reported from there. The limit for many species appears to be in the Cape Hatteras region—where the Carolinian and Virginian provinces meet. The faunal differences which characterize these two biogeographic provinces are accompanied by hydrographic differences in the water masses that lie to either side of the Cape.

To our discussion, the most important physical difference between these water masses is their temperature, for it is the probable cause for the limitation of many species in the Cape region. Temperature is the primary factor determining the distribution of marine organisms, provided substrate and salinity are not limiting. Neither substrate nor salinity are involved here, and the temperature differences are adequate to account for the observed limits to distribution. The temperature regime north of Cape Hatteras fits the requirements for a temperate zone as defined by Vaughan (1940), i.e. ranging from 10° to 25°C. South of Cape Hatteras, the temperature regime fits Vaughan's requirements for a subtropical zone, i.e. ranging from 15 to 30°C. Parr (1933) has shown that a "temperature barrier" exists in the Cape region in winter, and Wells and Gray (1960) have shown that a "temperature barrier" exists here in summer.

While temperature has usually been considered the primary factor in the distribution of sponges, Burton (1930b) has suggested that currents play this role. He traced the distribution of several transoceanic species and related them to dominant ocean currents. Currents flow along the coast toward North Carolina from both north and south, converge in the Hatteras region,

and flow northeastward from this point. The presence of northern and southern species in the Cape region can be attributed in part to the transport provided larvae by these currents. Because weather-induced currents occasionally do pass around the cape, they can carry larvae beyond this point, and provided suitable environmental conditions, sponges could develop beyond the limit of their previous geographical range. However, unless suitable temperature conditions permit its survival, this sponge will still be effectively limited by the Cape.

There is evidence that such interchanges of water do occur in the Hatteras area (Bumpus & Pierce 1955). However, the continuing limitation of many species in this area, in spite of this occasional interchange, indicates that temperature, and not current, is the limiting factor to their further distribution along this coast.

Dana (1853) and many other workers defined the region lying between Cape Cod and Cape Hatteras as the Virginian biogeographic province, and the region lying between Cape Hatteras and Cape Canaveral as the Carolinian province. While Ekman (1953) treats the Virginian province as being inhabited by a warm-temperate fauna, he considers the Carolinian province to consist of a mixture of warm-temperate and subtropical species. On the basis of their studies of intertidal organisms on this coast, Stephenson and Stephenson (1954) regard the Carolinian province to be warm-temperate and the Virginian province to be a transitional zone between the Carolinian province and the Acadian province north of Cape Cod, a "cold-temperate" province.

In terms of its sponge fauna, the Virginian province is low in endemic species and relatively low in total species. It is probably primarily transitional in nature, or a southern outlier of a northern or boreal fauna. The Carolinian province is relatively low in endemic species but relatively high in total species. It is apparently a subdivision of the West Indian Region, of a subtropical nature with offshore reefs inhabited by a tropical fauna.

This evaluation generally agrees with the Stephenson's analysis of Atlantic coast biogeographic subdivisions. They assign a similar rank to these provinces, but place them in different temperature zones, using a different set of definitions (those of Stephenson 1947). When treating these sponges of Bermuda, de Laubenfels (1950b) considered that area to be at the northern edge of the West Indian Region. In view of the predominance of West Indian species represented in North Carolina sponge collections, we also regard North Carolina as being on the northern edge of the West Indian Region.

By this reasoning, Cape Hatteras is not only a dividing point between the Virginian and Carolinian provinces, but also between a greater Atlantic Boreal Region to the north and a greater West Indian Region to the south, which is primarily tropical in nature. In 1896, Ortman recognized Cape Hatteras as a major dividing point between an Atlantic Boreal Region and an East American Region, which extended as far south as Brazil. Our study of North Carolina sponges supports this view.

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SUMMARY

1. A total of seventy species of sponges from coastal North Carolina have been described.
2. Of these, six are new to science, and thirty-two are new records for North Carolina.
3. In an analysis of distribution, fifty-five species apparently reach their distribution limits in North Carolina, temperature gradients in the Cape Hatteras region apparently being responsible for this limitation.
4. The sponge fauna of North Carolina is principally West Indian in affinity with a much smaller representation of northern species and widespread Atlantic coast species.
5. The proportional representation of southern species is greatest in collections made farthest south along the North Carolina coast, while the representation of northern species is greatest in collections made farthest north.

6. The Virginian Province is apparently a transition zone between the Carolinian and Acadian Provinces, or a temperate outlier of a northern Atlantic Boreal Region.
7. The Carolinian Province is apparently a subdivision of the West Indian Region, of subtropical nature, with tropical offshore reefs.

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MIMICRY IN THE COLOR PATTERN OF CERTAIN APPALACHIAN SALAMANDERS

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In 1927 Dunn described *Desmognathus fuscus imitator* on the basis of the presence of bright red cheek patches. This form was relegated to the synonymy of *Desmognathus fuscus carolinensis* (now *Desmognathus ochrophaeus carolinensis*) by Pope (1928) but the question of the similarity of this form to the sympatric *Plethodon jordani jordani* has remained of great interest. Furthermore, a sub-species characterized by red legs, *Plethodon jordani shermani*, occurs in the Nantahala Mts. where *carolinensis* has been reported with red legs (Brimley, 1928; Bishop, 1947). In addition to the presence of the red color in some individuals, the populations of *carolinensis* from the higher altitudes also resemble *jordani* in being somewhat darker than those from lower altitudes (Hairston, 1949). Neither of the above mentioned cases of apparent mimicry has been carefully reviewed to establish its merit as an example of mimicry. The present discussion is an attempt to provide such a review and to add a certain amount of data to the information available.

With respect to the *jordani*-"*imitator*" relationship, Dunn (1927) stated that the resemblance "seems to be a clear case of mimicry", but he advanced no hypothesis to account for it.

Brimley (1944) had the following comments on the subject: "Considering, however, that this form has only been reported from the region wherein either the red-cheeked *Plethodon jordani* or the redlegged *Plethodon shermani* occur it may be looked upon as a manifestation in this species (*carolinensis*) of the same causes which have produced the red cheeks of the one and the red legs of the other, or perhaps all three may simply exhibit traces of an old color pattern which has long since disappeared in their relatives but still persists in the two named species of *Plethodon* and turns up once in a while in this form also."

Hairston (1949) briefly discussed the problem but drew no conclusions. However, he did call attention to the remarkable fact that large percentages of *imitator* individuals occur only in populations which are sympatric with a brightly marked subspecies of *Plethodon jordani*. Whereas the frequency of yellow cheeks in populations of *Desmognathus ochrophaeus carolinensis* from the Blue Ridge Mountains, the Black Mountains, and the lower elevations of the Great Smoky Mountains averages only five per cent, about one-fourth of the population from above 5000 feet in the Great Smoky Mountains show the red or yellow color. Almost all of the *Plethodon jordani jordani* show it (King, 1939). This suggests that the distribution of colorbearing individuals is not due to random mutation alone. If this coloration were truly neutral, both *jordani* and *carolinensis* would be polymorphic with the number of individuals exhibiting the red color in a population at equilibrium fixed by the mutation rates, which are probably of the same order of magnitude. Except in very small populations or ones under exceptional mutation pressures, the frequency of occurrence of any one mutant can increase toward universality only where the mutation has positive survival value, directly or indirectly.

Noble (1931) discussed the problem and concluded that the reddish color had no apparent survival value. He suggested that a possible explanation might be that although the color was of no adaptive value *per se*, it was the by-product of some unknown physiological mutations having such value. However, even if the red cheek pattern (or red leg pattern) is assumed to be pleiotropic in nature, this does not explain why it appears to benefit *carolinensis* only in those particular regions where a colored subspecies of *jordani* occurs rather than over a larger portion of its range. Nor does it explain the exact parallelism with *jordani* and *shermani* in both cases.

If, on the contrary, one assumes that the coloration has survival value, the benefit must fall into one of the following four categories:

1. *Advantageous with respect to the physical environment*—This explanation would assume that some non-biological characteristic of the habitat of the higher elevations of the Southern Appalachian Mountains such as rainfall, humidity, temperature, or a combination of such factors, makes by itself the presence of red coloration advantageous to a salamander. Since we can

never know everything about the relations between an animal and its physical environment, we can not eliminate this theory, but it is difficult to envisage such a relation as more than remotely probable

2. *Advantageous with respect to intraspecific relations*—This explanation would assume that the pattern enables the salamanders to recognize one another, *e.g.*, during the mating season. This is highly unlikely for there is no sexual dimorphism with respect to the color patterns and these animals apparently are color blind as one would expect of a species which is principally nocturnal in habits (Polyak, 1955). Hence the flashy red would be unnecessary and indeed all evidence points to olfactory and tactile recognition in these types (Noble, 1931).

3. *Advantageous with respect to prey*—Salamanders play an important part in the food chains of the community in which they live, both as predator and prey (Hairston, 1949). The diet of these species presumably consists in large part of insects, some of which at least are color sensitive. That the red color near the head of *jordani* attracts insects so that they may be more readily captured is a plausible explanation. The facts that the rear legs of *shermani* are also red and that the red coloration of *carolinensis* occurs only in the regions where red-colored plethodons occur, rule against this theory in favor of the next.

4. *Advantageous with respect to predators*—Such coloration would be either cryptic or aposematic. Brattstrom (1955) recently proposed that the so-called warning coloration of coral snakes was actually cryptic (disruptive) in nature. Later workers (Hecht and Marien, 1956; Mertens, 1956) have concluded that although the pattern may serve in a cryptic manner, especially in the case of mimics, the principal function of the pattern is as a warning device. It would seem that the case of these salamanders is parallel in nature. Although the coloration might serve a deceptive or illusory purpose, it is again difficult to see why the "*imitator*" forms occur only in those areas where *jordani* or *shermani* occurs.

The model-mimic explanation would assume the presence of some undesirable quality in the model (*jordani*) and a mimetic adaptation on the part of the mimic ("*imitator*"). If this theory is correct, then at least one of the predators from which the model is protected by its aposematic coloration is probably color-sensitive, especially to red and yellow. This does not mean that although the color is of specific value with respect to color-sensitive predators only, the *pattern* is not of some value with respect to color-blind predators. In fact, it might well be that the latter are of greater importance and that the color is a refinement for the color-sensitive animal.

Assuming Theory Four to be correct, the following experimental work was done in order to to test the hypothesis. The author is aware that such experiments have been severely criticized by authors who do not subscribe to the mimicry theory (McAtee, 1912), but in lieu of actual field observations they are the best available sources of data.

The present series of experiments does not compare in thoroughness with the excellent study of mimicry in butterflies by Brower (1958 a, b, c). However, because of the limited number of salamanders available the present experiments could not be carried out over a large number of trials. Brower (1958 a) estimated that about 1000 butterflies were used in her series with one model-mimic pair. In contrast, only 60 salamanders could be obtained for the present study and these were distributed among several predators. Thus the experiments were not extended to the point where the predators learned to recognize the model. However, in view of the fact that it has been definitely established that recognition of this type can occur, the importance of the present experiments lies in establishing the presence or absence of a distasteful factor in *jordani*.

A series of *Plethodon j. jordani* was collected from near Indian Gap and Mt. Collins in the Great Smoky Mountains. At the same time a series of *Desmognathus* was collected in various parts of the Southern Appalachian Mountains. It might have been preferable to use *carolinensis* from near the type locality of *imitator* (Indian Gap) but permission to collect this species in the Great Smoky Mountains National Park was not obtained. The majority of specimens were collected just north of the park boundary, two to three miles east of Gatlinburg, Tennessee.

Three species of animals were used as predators. The first was a common garter snake, *Thamnophis sirtalis*, which is one of the few reptiles reaching elevations above 5000 feet where *jordani* is abundant (King, 1939). The second predator was an adult male sparrow hawk, *Falco sparverius*. Although it occurs in the Great Smoky Mountains (Wetmore, 1939), it is unlikely that the latter species is a natural predator of these

salamanders. It was chosen because of availability and because it should be fairly typical of bird predators in general. The third type of predator was a pair of migrant shrikes, *Lanius ludovicianus*. They, too, were chosen because of availability.

In addition to the above, an adult skunk, *Mephitis mephitis*, a young raccoon, *Procyon lotor*, and two other male sparrow hawks were offered salamanders, but refused to eat any and thus could not be used as test animals.

Garter Snake—Extensive tests were not run with this predator but it was offered two specimens of *P. j. jordani*. It ate both, evidencing no signs of distaste, although it did acquire a certain amount of slime on its head from the salamanders. Hence it appears that there is no "undesirable" factor in *jordani* as far as the garter snake is concerned.

Hawk—The hawk was offered *jordani* and *carolinensis* approximately alternately. It rejected the first two *jordani* which it attacked. In both cases it attacked the salamander vigorously, seizing it with feet and beak on the dorsum. Almost immediately it dropped the victim and returned to its perch against which it vigorously rubbed its beak. It had been observed to exhibit a similar reaction when it was offered a piece of meat which had started to spoil. This might indicate an unpleasant taste. The salamanders, although wounded and temporarily dazed, survived for a period of a week in apparent good health, at the end of which time they were killed and preserved. The hawk never refused a *carolinensis*. It would often tear one apart and eat it piecemeal.

Toward the end of the experiments which covered a period of about ten days, the hawk exhibited signs of increased hunger, since it had been fed little save salamanders. It was at this time that the hawk accepted the *jordani*. Its behavior was quite different from that exhibited earlier, however. It attacked only the head and always ate the victim whole. The fact that a predator accepts certain prey when extremely hungry does not necessarily mean that the prey is "desirable" under normal circumstances (Cott, 1940).

The preliminary experiments with the hawk showed the desirability of controlling, insofar as was possible, such variables as the hunger of the predator, the conditions of testing, *etc.* Therefore some effort was made to improve the experimental conditions.

TABLE 1

Species offered	No. attacked	No. eaten	No. refused	% refused
GARTER SNAKE				
P. j. jordani	2	2	0	0%
SPARROW HAWK:				
P. j. jordani	7	5	2	28%
D. o. carolinensis	4	4	0	0%

Shrike—The hunger of the birds was kept within reasonable limits by feeding the birds to capacity at 8:00–9:00 am and beginning the tests between 12:00 m and 6:00 pm. Experiments were continued at approximately two-hour intervals until 10:00–11:00 pm. If after testing the birds were hungry, they were again fed. The birds were fed on liver with some meal worms and other insects occasionally.

The experimental area consisted of a glass covered table, 30″ x 48″. Along one of the long sides of the table was a perch extending 18″ above the base. The table was illuminated by a 150 W Westinghouse projector floodlamp placed about 5 feet above the table. The remainder of the test room was in semi-darkness which reduced distraction of the predator by the observer. In conducting each trial, the prey was placed in the center of the table and then the predator was placed on the perch. The following categories were used to classify the reactions of the predator: 1) *No attack* (NA)—The predator failed to attack within five minutes. 2) *Prey escaped* (PE)—Although the predator attacked, the prey succeeded in getting off the table before being killed. 3) *Rejection after attack* (R)—The predator attacked, but voluntarily released the prey and ignored it thereafter. If the predator dropped the prey but returned to it within one minute, the test was continued and this category was not indicated. This category could be further subdivided into those victims which survived (RL) and those which died (RD). 4) *Prey killed* (K)—The prey was attacked, killed, and usually eaten completely.

It was realized that there was no clear-cut boundary between groups 2 and 3. If the predator paused near the edge of the table and the prey leaped off, was this rejection or escape? Fortunately this happened only once.

The test animals were arbitrarily divided into two groups on the basis of size with 70 mm as the critical length. The sequence of presentation

TABLE 2

Results of tests with shrikes; corrected for frequency

Class	NA	PE	RL	RD	K	Total Survival (NA, PE, RL)
d	8% (0–1)	0% (0–0)	0% (0–0)	8% (0–1)	84% (2–8)	8%
D	20% (0–2)	10% (1–0)	10% (0–1)	20% (0–2)	40% (2–2)	40%
p	29% (0–2)	0% (0–0)	14% (1–0)	14% (0–1)	43% (1–2)	43%
P	7% (0–1)	13% (0–2)	60% (3–6)	13% (0–2)	7% (0–1)	80%

of prey was determined randomly both as to species and size. These sequences are given under the discussion of each test animal. Capital letters represent specimens greater than 70 mm; lower case, less than 70 mm. Genera are designated by their initial letter.

The first shrike used (S-1) readily ate salamanders. It was offered *Desmognathus* and *Plethodon* in the following sequence: dpPDDpPPDd. The results are listed in Table 2 (the first number in parentheses refers to S-1). The only *jordani* eaten was the first one offered, a small one. The next *jordani* offered was a large specimen which lashed about with its tail when attacked. Almost immediately the shrike released the prey, flew straight up about four feet, and then flew in circles about the room until it flew into a wall. Upon examination, it was found that both eyes were partially blinded by feathers sticking tightly to the nictitating membrane. For all practical purposes the bird was blind, although it could distinguish light and dark. It was helpless and, depending on how long the condition lasted, the results might have proved fatal under natural circumstances. In subsequent tests the bird attacked *jordani* more cautiously and never received as heavy an amount of slime.

The second shrike used (S-2) did not accept salamanders as readily as S-1, but it was possible to extend the experiments with it over a longer period of time. It was offered prey in the following sequence: d D d P P d P P p d D P d p D D P d P d D P P d P p d p P P D D P d. The results are listed in Table 2 (Second number in parentheses refers to S-2). This bird never had as drastic an experience with slime as S-1, but resembled it in behavior otherwise. The behavior of the desmognaths has been previously reported (Huheey, 1959). The main difference between the two genera was that the plethodons were less agile and relied more on slime, while the desmognaths relied on agility and autotomy.

In order to compensate for the disparity in size of the four classes (d, D, p, P), the results of the experiments with the two shrikes have been combined and expressed as per cent (see Table 2). Although the results of the experiments could be expressed statistically in numerous ways, the trend is obvious from the table and only one test of significance will be given here. Using the method of χ^2, the probability that the results with respect to the survival rate of *Plethodon* versus *Desmognathus* are due to chance alone is slightly greater than 0.001 ($\chi^2 = 9.2$, n = 1).

From the above it can be concluded that *carolinensis* is relatively unprotected as far as distastefulness (to the predators used) is concerned. On the other hand, *jordani* has a certain amount of protection of this nature, the effectiveness of which depends somewhat upon the nature, condition, and behavior of the predator. This protection is of considerable survival value since most specimens, even some which were injured, survived.

After the data from the hawk had been obtained, it was felt that perhaps the repellent factor lay in some unpleasant taste residing on the skin of the salamanders, although they proved tasteless to humans. Observations on the shrike, however, have convinced the writer that the "distasteful factor" is the slime which is exuded copiously from the tail of the larger specimens, and to a lesser degree, the entire body of all specimens. The "sliminess" of *jordani* has been a matter of some difference of opinion. Some authors have stated that it was only slightly slimy. King (1939) states, however, that they are quite slimy and that the slime is extremely hard to remove. The author has found adult specimens to be fully as slimy as *glutinosis*, some individuals even exceeding the latter in the rate of exudation of slime.

The evidence that *jordani* has a "distasteful factor" in or on its skin makes the theory that

the red cheek color is aposematic in function seem the most reasonable of all so far advanced. The resulting mimicry by *carolinensis* (pseudaposematic coloration) is therefore also likely. It must be admitted that the undesirability in *jordani* is not 100% and that the efficiency of the warning coloration is probably not 100% in *jordani* and even less effective with respect to "*imitator*". However, it should be remembered that natural selection is not an "all-or-nothing" affair; even slight advantages will in time become firmly established (Dobzhansky, 1951).

It seems pertinent to point out that *carolinensis* is more nearly like *jordani* in its ecological requirements than any other *Desmognathus*. Only it and *wrighti* are truly terrestrial and the latter is specialized for a different niche. It may be argued that *carolinensis* prefers a wetter habitat than *jordani*, and my observations have tended to bear this out. However, I have found both species in the same log. In addition, the spatial difference between a relatively dry "*jordani*" log and a wet "*carolinensis*" seep may be only a few feet. This would be insignificant insofar as a mobile predator were concerned.

Another objection to the interpretation in terms of warning coloration is that since these animals would presumably be abroad only at night, their warning coloration would fail to protect them under the poorer light conditions. However, these same arguments have been proposed (Brattstrom, 1955) and rejected (Mertens, 1956; Hecht and Marien, 1956) in the case of the coral snake. In the first place, these animals are probably not *exclusively* nocturnal, and more important, they are quite likely to be exposed by foraging animals in the daytime.

Desmognathus ochrophaeus carolinensis has not succeeded in achieving consistent mimicry presumably because of several possible factors: 1) Of necessity it had to evolve its pattern after its model was well on its way toward establishing its pattern. Perhaps the mimicry involved here is of comparatively recent origin and not completely developed yet. 2) Unlike *jordani*, *carolinensis* is not restricted to the mountain tops and there is gene exchange with populations from lower altitudes which derive no benefit from such coloration, and indeed, show it less frequently (King, 1939). 3) A Batesian mimic receives protection inversely as the ratio of its abundance to that of its model, and if dimorphic, will reach equilibrium at frequencies such that both forms receive equal protection (Fisher, 1958; Sheppard, 1959). Such dimorphism is well known in butterflies.

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SUMMARY

Various theories as to the origin and function of the color patterns of *Plethodon jordani jordani*, *Plethodon j. shermani*, and sympatric populations of *Desmognathus ochrophaeus carolinensis* are examined and discussed. It is concluded that the most plausible explanation of the color patterns is that they serve aposematic and pseudaposematic functions. The experimental data here reported agreed with this conclusion. To various predators, *jordani* has a lower degree of acceptability than *carolinensis*. The absence of the universality of the red color in *carolinensis* is explained in terms of less effective selective pressures and gene exchange with populations of the lower altitudes.

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HELLINGER INTEGRALS IN INNER PRODUCT SPACES[1]

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Prefatory Remarks

The ensuing paper was written in 1955, publication being withheld because of its unusual length coupled with crowded conditions in three journals to which the paper was submitted. In the intervening period, some of the ideas in Chapters 2 and 3 have been amplified in a more abstract setting by other researchers (see, for instance, G. B. Pedrick, "Theory of Reproducing Kernels in Hilbert Spaces of Vector Valued Functions," University of Kansas Studies in Eigenvalue Problems, Technical Report 19, 1957): those ideas were originally reported at a meeting of the American Mathematical Society in December, 1954, by the present author [8].

The final decision to publish is dictated in part by requests for references to these results in the literature, and in part by the recent discovery, made by James R. Webb at the University of Texas (Abstract 568-8, Notices of the American Mathematical Society, VII (2), April 1960, p. 245), of a remarkable theorem in which Hellinger integrals are used to characterize the continuous linear functionals on the space of quasi-continuous functions from a number interval. The present paper appears here in its original form, and acknowledgment is hereby made of financial assistance from the University Research Council in the form of publication subsidy.

Introduction

By an *inner product space* we mean an ordered pair $\{S, Q\}$ such that S is a (complex) linear space and Q is an inner product for S; *limits* in $\{S, Q\}$ (*i.e.*, in S with respect to Q) are understood to be defined with respect to the norm given by $\| x \| = Q(x, x)^{1/2}$ for x in S; the space is *complete* provided it is true that each convergent (*i.e.*, "Cauchy convergent" or "fundamental") sequence in $\{S, Q\}$ has a limit in $\{S, Q\}$.[2] For an inner product space $\{S, Q\}$, we use $B\{S, Q\}$ to denote the space of continuous linear transformations T in $\{S, Q\}$, with norm given by $| T | =$ L.U.B. $\| Tx \|$ for $\| x \| = 1$. Since we regard $B\{S, Q\}$ as "containing" the complex numbers then T^*, the adjoint of T in $B\{S, Q\}$, is the complex conjugate of T if T is a complex number. Throughout this paper, $\{S_0, Q_0\}$ is assumed to be a complete inner product space.

In 1907, E. D. Hellinger [5] introduced certain integrals which now bear his name and which are limits—of approximating sums—through the process of successive refinement of subdivisions of a number interval. One of these integrals, namely

$$(1) \qquad \int_0^1 \frac{[df][dg]^*}{d\phi}, \quad f \text{ and } g \text{ complex}, \quad \phi \text{ real and non-decreasing},$$

assumed a central role in the theory of quadratic forms in infinitely many variables [6]. The equivalence between the integral (1) and a certain Lebesgue integral (of the form $\int_0^1 f_1 g_1^* d\phi$ where f and g are "indefinite" integrals of f_1 and g_1, respectively) was first established by H. Hahn [4].

There remained many other connections between the theory of Hellinger's integrals and the theory of positive definite quadratic forms in the Hilbert space of number sequences $\{x_n\}$ with $\sum | x_n |^2$ convergent—especially with respect to limiting processes involved and functions generated by these processes. Such connections seem to have provided substantial stimulus for much of E. H. Moore's general analysis theory [11], and for the general theory of limits developed by E. H. Moore and H. L. Smith [12]. The importance of D. Hilbert's theory of linear integral equations [7] as background to all of these ideas, and to those which follow, cannot be emphasized too strongly.

We extend the idea of the Hellinger integral (1) to obtain an integral (Definition 1.1) which we denote by

$$(2) \qquad \int_0^1 Q_0(df, [d\phi]^{-1} dg),$$

[1] Presented, in six parts, to the American Mathematical Society on August 30, 1955.

[2] Thus, we have the postulates A, B, and E for a Hilbert space as originally defined by J. von Neumann [13]. We assume the elementary properties of complete inner product spaces, and refer the reader to von Neumann [14] for pertinent concepts.

where each of f and g is a function from [0, 1] to S_0 and the function ϕ, from [0, 1] to $B\{S_0, Q_0\}$, is such that $\phi(b) - \phi(a)$ is Hermitian and non-negative for each subinterval $[a, b]$ of [0, 1]. This extension from the one-dimensional case (where $B\{S_0, Q_0\}$ is merely $\{S_0, Q_0\}$) has suggested two main directions for our investigation.

One direction is toward a study of the linear[3] space S of functions f from [0, 1] to S_0 such that $\int_0^1 Q_0(df, [d\phi]^{-1}df)$ exists, and connections between this space and a system of Stieltjes integral equations of the form

$$(3)\qquad \begin{cases} f_1(b) - f_1(a) + \displaystyle\int_a^b d\phi \cdot f_2 = 0 \text{ and} \\ \displaystyle\int_a^b d\theta \cdot f_1 + f_2(b) - f_2(a) = h(b) - h(a) \end{cases}$$

for each subinterval $[a, b]$ of [0, 1].

The integrals in the system (3) are the mean integrals (as Hellinger-type limits) previously studied by the author: solutions are furnished by the author's extensions ([9] and [10]) of H. S. Wall's theory of harmonic matrices [15]. With suitable restrictions on ϕ and θ, this space S has an inner product Q of the form

$$(4)\qquad Q(f, g) = \int_0^1 Q_0(df, [d\phi]^{-1}\, dg) + \int_0^1 Q_0(f, d\theta \cdot g).$$

Various boundary value problems connected with the system (3) determine certain subspaces of the space $\{S, Q\}$, and lend themselves to classification in terms of relationships among the associated subspaces. These boundary value problems lead to integral equation problems similar to those arising from the classical Sturm-Liouville theory of second order differential equations with two-point boundary conditions (see, for example, Chapter 5 of [3]).

Another direction is toward a study of complete inner product spaces $\{S, Q\}$ of functions from an arbitrary set E to $\{S_0, Q_0\}$—spaces which are related to the space described in the preceding paragraph in that convergence in $\{S, Q\}$ implies point-wise convergence over E. Such a space is characterized by the existence of a function K (a *kernel* in the space $\{S, Q\}$—Definition 2.1) from $E \times E$ to the space of linear transformations in S_0 such that, for each t in E and each x in S_0, the function[4] $K(\cdot, t)x$ is in S and

$$(5)\qquad Q(f[\cdot], K[\cdot, t]x) = Q_0(f[t], x)$$

for all f in S.

These kernels, in turn, are characterized by the property that

$$(6)\qquad \sum_{i,j} Q_0(x_i, K[t_i, t_j]x_j) \geq 0$$

for each finite sequence $\{t_i, x_i\}$ in $E \times S_0$. In the case that S_0 is one-dimensional, these kernels are the *positive Hermitian matrices* of E. H. Moore's general analysis; also, they are the functions studied by N. Aronszajn as *reproducing kernels*. The reader is referred to Aronszajn's paper [1] for an historical summary and for the main theorems in the one-dimensional case: there one also finds an extensive bibliography.

There is a strong interplay between these two main directions of our investigation—the former providing valuable examples for the latter, and the latter supplying powerful tools for the former. For this reason we have elected to present our report in one paper, although some of our results have been announced separately ([8], abstract). Subsequent sections of this paper are called chapters.

Chapter 1 is devoted primarily to the integral (2), the development being based on a fundamental inequality (Lemma 1.1):

$$(7)\quad Q_0([x + y], [H + K]^{-1}[x + y]) \leq Q_0(x, H^{-1}x) + Q_0(y, K^{-1}y),$$

where H and K are non-negative Hermitian elements of $B\{S_0, Q_0\}$, x is in $H(S_0)$, y is in $K(S_0)$, and $x + y$ is in $[H + K](S_0)$. In the next two chapters we treat general kernels and relationships between *kernel systems*, sequences $\{K, E, S, Q\}$ such that K is the kernel in the inner product space $\{S, Q\}$ of functions from the set E to $\{S_0, Q_0\}$. In Chapter 4 we consider *ideal* kernel systems (Definition 4.1): all systems are ideal if S_0 is finite dimensional, but otherwise there are always two possibilities; in an ideal kernel system $\{K, E, S, Q\}$, Q is a Hellinger integral over E (Theorem 4.4). Chapters 5 and 6

[3] The "linearity" of a space of functions always refers to the point-wise addition and numerical multiplication which are indicated by the context.

[4] The notation $K(\cdot, t)x$ describes the function k from E to S_0 defined by the equation: $k(s) = K(s, t)x$ for s in E. This convention goes back at least to Hilbert [7] and we use it, as an abbreviation, whenever it seems convenient.

are devoted to the system (3) and problems arising from it. In the seventh (and final) chapter, we show how our ideas are applicable in function theory in such a way as substantially to extend many of the ideas of S. Bergman [2] and others.

1. The first integral

We suppose, in this chapter, that ϕ is a function from the number interval [0, 1] to $B\{S_0, Q_0\}$, that $\phi(0) = 0$, and that ϕ is *non-decreasing* in the sense that, for each subinterval $[a, b]$ of [0, 1], $Q_0(x, [\phi(b) - \phi(a)]x) \geq 0$ for all x in S_0, *i.e.*, $\phi(b) - \phi(a)$ is Hermitian and non-negative. A *subdivision* of [0, 1] is an increasing number sequence $s = \{s_i\}_0^n$ such that n is a positive integer and $[s_0, s_n]$ is [0, 1]; a *refinement* of s is a subdivision t of [0, 1] of which s is a subsequence.

Definition 1.1 Suppose each of f and g is a function from [0, 1] to S_0 and, for each subinterval $[a, b]$ of [0, 1], each of $f(b) - f(a)$ and $g(b) - g(a)$ is a point in $[\phi(b) - \phi(a)](S_0)$. The *Hellinger integral* $\int_0^1 Q_0(df, [d\phi]^{-1}dg)$ is a number J such that for each positive number c there is a subdivision s of [0, 1] such that if t is a refinement of s then $|J - \sum_t Q_0(df, [d\phi]^{-1}dg)| < c$—for $t = \{t_i\}_0^n$, $\sum_t Q_0(df, [d\phi]^{-1}dg)$ denotes the number

$$\sum_{i=1}^{n} Q_0([f(t_i) - f(t_{i-1})], [\phi(t_i) - \phi(t_{i-1})]^{-1}[g(t_i) - g(t_{i-1})]),$$

where (for each i) $[\phi(t_i) - \phi(t_{i-1})]^{-1}[g(t_i) - g(t_{i-1})]$ denotes any point x of S_0 such that $[\phi(t_i) - \phi(t_{i-1})]x = g(t_i) - g(t_{i-1})$.

Lemma 1.1 *If each of H and K is a non-negative Hermitian element of $B\{S_0, Q_0\}$ then*

$$(1.1)\quad Q_0([x + y], [H + K]^{-1}[x + y]) \leq Q_0(x, H^{-1}x) + Q_0(y, K^{-1}y)$$

for all x and y in S_0 for which these symbols have meaning in the sense that $H^{-1}x$ is any point u of S_0 such that $Hu = x$, etc.; if, moreover, each of H and K is positive (i.e., reversible) then "=" holds in (1.1) only in case $H^{-1}x = K^{-1}y$.

Proof. Let $x = Hu$, $y = Kv$, and $x + y = [H + K]w$. Then $K[v - w] = H[w - u]$, and the lemma follows from the computation:

$$\begin{aligned} &Q_0([u - w], H[u - w]) + Q_0([v - w], K[v - w]) \\ &= Q_0([u - v], H[u - w]) \\ &= Q_0(u, H[u - w]) + Q_0(v, K[v - w]) \\ &= Q_0(u, Hu) + Q_0(v, Kv) - Q_0(Hu + Kv, w) \\ &= Q_0(Hu, u) + Q_0(Kv, v) - Q_0([H + K]w, w) \\ &= Q_0(x, H^{-1}x) + Q_0(y, K^{-1}y) \\ &\qquad - Q_0([x + y], [H + K]^{-1}[x + y]). \end{aligned}$$

Lemma 1.2 *If H is a non-negative Hermitian element of $B\{S_0, Q_0\}$ and y is a point of $H(S_0)$ then*

$$(1.2)\quad Q_0(y, y) \leq |H| Q_0(y, H^{-1}y),$$

where $H^{-1}y$ denotes any point x in S_0 such that $Hx = y$.

Proof. This lemma is a consequence of Schwarz's inequality for the non-negative Hermitian form $Q_0(x, Hx)$:

$$\begin{aligned} Q_0(Hx, Hx)^2 &\leq Q_0(Hx, HHx)\, Q_0(x, Hx) \\ &\leq |H| Q_0(Hx, Hx)\, Q_0(x, Hx). \end{aligned}$$

Lemma 1.3 *If $\{H_n\}$ is a sequence of Hermitian elements of $B\{S_0, Q_0\}$ and for each x in S_0 $\{Q_0(x, H_n x)\}$ is a bounded non-decreasing number sequence, then there is an element K of $B\{S_0, Q_0\}$ such that for each x in S_0 the sequence $\{H_n x\}$ has the limit Kx in $\{S_0, Q_0\}$.*

Proof. For x and y in S_0 and positive integers i and j such that $j < i$ we have $H_i - H_j$ non-negative and

$$\begin{aligned} &|Q_0(x, [H_i - H_j]y)|^2 \\ &\leq Q_0(x, [H_i - H_j]x)\, Q_0(y, [H_i - H_j]y). \end{aligned}$$

It follows that there exists a linear transformation K from S_0 to S_0 such that for each x and y in S_0 the sequence $\{Q_0(x, H_n y)\}$ has the limit $Q_0(x, Ky)$. We see that K is Hermitian and, since $\{S_0, Q_0\}$ is complete, K belongs to $B\{S_0, Q_0\}$. Since, for each positive integer n, $K - H_n$ is non-negative and $|K - H_n| \leq |K - H_1|$ and thus

$$\begin{aligned} &Q_0([K - H_n]x, [K - H_n]x) \\ &\leq |K - H_1| Q_0(x, [K - H_n]x) \end{aligned}$$

for all x in S_0 (by Lemma 1.2), the lemma is established.

Theorem 1.1 *If f is a function from [0, 1] to S_0 such that $f(b) - f(a)$ belongs to $[\phi(b) - \phi(a)](S_0)$ for each subinterval $[a, b]$ of [0, 1], then $\int_0^1 Q_0(df, [d\phi]^{-1}df)$ exists only in case there exists a non-negative number N such that for each subdivision t of [0, 1]*

$$(1.3)\quad \sum_t Q_0(df, [d\phi_0]^{-1}df) \leq N,$$

in which case $\int_0^1 Q_0(df, [d\phi]^{-1}df)$ is the least such N.

This is a direct application of Lemma 1.1,

which shows that our approximating sums are non-decreasing with successive refinements of a subdivision of [0, 1].

THEOREM 1.2 *If f is a function from [0, 1] to S_0 such that $f(b) - f(a)$ belongs to $[\phi(b) - \phi(a)](S_0)$ for each subinterval $[a, b]$ of [0, 1], then $\int_0^1 Q_0(df, [d\phi]^{-1}df)$ exists only in case there exists a non-decreasing function h from [0, 1] to the real numbers such that for each subinterval $[a, b]$ of [0, 1]*

$$Q_0([f(b) - f(a)], [\phi(b) - \phi(a)]^{-1}[f(b) - f(a)]) \leq h(b) - h(a). \tag{1.4}$$

The sufficiency of (1.4) follows from the preceding theorem, and the necessity follows by considering the function h such that $h(0) = 0$ and $h(b) = \int_0^b Q_0(df, [d\phi]^{-1}df)$ for b in [0, 1]. We observe that, by Lemma 1.2, (1.4) implies

$$Q_0([f(b) - f(a)], [f(b) - f(a)]) \leq |\phi(b) - \phi(a)||h(b) - h(a)|; \tag{1.5}$$

this observation leads immediately to

THEOREM 1.3 *If f is a function from [0, 1] to $\{S_0, Q_0\}$ and $\int_0^1 Q_0(df, [d\phi]^{-1}df)$ exists then f is quasi-continuous, i.e., the limit $f(r+)$ exists for each r in [0, 1) and the limit $f(r-)$ exists for each r in (0, 1]; moreover, f is continuous if ϕ is continuous, and f is of bounded variation if ϕ is of bounded variation.*[5]

Proofs of the remaining four theorems of this chapter are readily constructed by direct computation and by application of preceding lemmas and theorems. Since more light is shed on them by the theorems of subsequent chapters, we omit our proofs.

THEOREM 1.4 *Suppose H is a non-negative Hermitian element of $B\{S_0, Q_0\}$, and S is the set of all functions f from [0, 1] to $\{S_0, Q_0\}$ such that $\int_0^1 Q_0(df, [d\phi]^{-1}df)$ exists and $f(0)$ belongs to $H(S_0)$. Then S is a linear space, and the equation,*

$$Q(f, g) = Q_0(f[0], H^{-1}g[0]) + \int_0^1 Q_0(df, [d\phi]^{-1}dg), \tag{1.6}$$

defines an inner product Q for S with the property that

$$Q_0(f[r], f[r]) \leq [\|H\|^{1/2} + |\phi(r)|^{1/2}]^2 Q(f, f) \tag{1.7}$$

for all f in S and all r in [0, 1].

THEOREM 1.5 *The inner product space $\{S, Q\}$ of Theorem 1.4 is complete provided that (i) the linear manifold $H(S_0)$ is closed in $\{S_0, Q_0\}$, and (ii) for each subinterval $[a, b]$ of [0, 1] the linear manifold $[\phi(b) - \phi(a)](S_0)$ is closed in $\{S_0, Q_0\}$.*

THEOREM 1.6 *Under the suppositions of Theorem 1.4, let K be the function from [0, 1] × [0, 1] defined by: $K(s, t) = H + \phi(s)$ if $s \leq t$ and $K(s, t) = H + \phi(t)$ if $t \leq s$. Then*

(i) if t is in [0, 1] and x is in S_0 then the function[6] $K(\cdot, t)x$ is in S and $Q(f[\cdot], K[\cdot, t]x) = Q_0(f[t], x)$ for all f in S.

(ii) if $t = \{t_i\}_0^n$ is a subdivision of [0, 1] and $\{x_i\}_0^n$ is a sequence in S_0 then the function $k(\cdot) = \sum_0^n K(\cdot, t_j)x_j$ is in S and has the property that, for $\{y_i\} = \{\sum_i^n x_j\}$,

$$\begin{cases} Q(k, k) = \sum_{i=0}^{n}\sum_{j=0}^{n} Q_0(x_i, K[t_i, t_j]x_j) \\ \qquad = Q_0(y_0, Hy_0) \\ \qquad + \sum_{i=1}^{n} Q_0(y_i, [\phi(t_i) - \phi(t_{i-1})]y_i). \end{cases} \tag{1.8}$$

THEOREM 1.7 *Suppose the function F, from [0, 1] to $B\{S_0, Q_0\}$, has the property that if x is a point of S_0 and $f(\cdot) = F(\cdot)x$ then $\int_0^1 Q_0(df, [d\phi]^{-1}df)$ exists. Then the integral $\int_0^1 [dF]^*[d\phi]^{-1}[dF]$ exists in the sense that there is an Hermitian element T of $B\{S_0, Q_0\}$ such that for each x in S_0 and each positive number c there is a subdivision s of [0, 1] such that if $t = \{t_i\}_0^n$ is a refinement of s and*

$$y = \sum_{i=1}^{n} [F(t_i) - F(t_{i-1})]^*[\phi(t_i) - \phi(t_{i-1})]^{-1} [F(t_i) - F(t_{i-1})]x$$

then $Q_0(y - Tx, y - Tx) < c$.

Illustration We now give a realization of the situation treated in this chapter. Let $\{S_0, Q_0\}$ be two-dimensional complex Euclidean space, identify $B\{S_0, Q_0\}$ with the collection of 2 × 2 matrices of complex numbers in the usual way, and define

$$\phi(u) = \begin{pmatrix} u & u^2/2 \\ u^2/2 & u^3/3 \end{pmatrix} \tag{1.9}$$

for u in the interval [0,1].

[5] As in [9] and [10], "bounded variation" is to be interpreted in terms of the appropriate norm on the space in which the functions have their values: *e.g.*, the total variation of f is $\int_0^1 \|df\|$, while that of ϕ is $\int_0^1 |d\phi|$.

[6] See footnote 4.

For each subinterval $[a, b]$ of $[0, 1]$ and each $x = x_1, x_2$ in S_0,

$$Q_0(x, [\phi(b) - \phi(a)]^{-1}x) = \frac{|x_1|^2}{b-a} + \frac{12}{b-a}\left|\frac{x_2}{b-a} - \frac{b+a}{2}\frac{x_1}{b-a}\right|^2. \tag{1.10}$$

From this it is readily found that if $f = f_1, f_2$ is a function from $[0, 1]$ to S_0 then, in order that $\int_0^1 Q_0(df, [d\phi]^{-1}df)$ exist, it is necessary and sufficient that (i) the ordinary Hellinger integral $\int_0^1 |df_1(u)|^2/du$ exist and (ii) $f_2(b) - f_2(a) = \int_a^b u df_1(u)$ for each subinterval $[a, b]$ of $[0, 1]$. If, in Theorem 1.4, we have

$$H = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \tag{1.11}$$

then the condition that f belong to S imposes the further restriction that (iii) $f_2(0) = 0$.

2. Kernels in inner product spaces

Suppose E is a set. If $\{S, Q\}$ is an inner product space of functions from E to the numbers, then a *kernel* in $\{S, Q\}$ is a function K from $E \times E$ to the numbers such that if t is in E then the function $K(\cdot, t)$ is in S and $Q(f[\cdot], K[\cdot, t]) = f(t)$ for all f in S (see, for example, [1]). We introduce the following extension of this idea.

Definition 2.1 The statement that K is a *kernel* in the inner product space $\{S, Q\}$ of functions from E to $\{S_0, Q_0\}$ means that K is a function from $E \times E$ such that (i) if s, t is in $E \times E$ then $K(s, t)$ is a linear transformation from S_0 to S_0 and (ii) if t, x is in $E \times S_0$ then $K(\cdot, t)x$ is in S and $Q(f[\cdot], K[\cdot, t]x) = Q_0(f[t], x)$ for all f in S.

Remark. The concept here defined is equivalent to the previous one in case S_0 is one-dimensional, *i.e.*, in case S_0 is the complex plane. Although we are dealing exclusively with complex spaces, slight modifications in theorems and proofs will yield analogous results for real spaces. Since multiplication of real quaternions may be realized as a matrix-times-vector multiplication in a real Euclidean space $\{S_0, Q_0\}$ of four dimensions, our results are easily modified further to apply to the idea of kernels in certain spaces of functions with real quaternion values—one merely restricts the values of K to a particular subset of $B\{S_0, Q_0\}$, *e.g.*, to real 4×4 matrices of the form,

$$\begin{pmatrix} d & -c & b & a \\ c & d & -a & b \\ -b & a & d & c \\ -a & -b & -c & d \end{pmatrix},$$

which corresponds to the vector $\{a, b, c, d\}$ and to the quaternion $ia + jb + kc + d$, where $i^2 = j^2 = k^2 = -1, ij = -ji = k, jk = -kj = i$, and $ki = -ik = j$. We shall make no further reference to such possible modifications of the present theory.

Theorem 2.1 *If K is a kernel in the inner product space $\{S, Q\}$ of functions from E to $\{S_0, Q_0\}$ then K has the property that:*

$$\sum_{i,j} Q_0(x_i, K[t_i, t_j]x_j) \geq 0 \tag{2.1}$$

for each finite sequence $\{t_i, x_i\}$ in $E \times S_0$.

Proof. The double sum in (2.1) is $Q(k, k)$, where k is the function $\sum K(\cdot, t_j)x_j$—a member of S by the linearity of S.

Theorem 2.2 *If K is a function from $E \times E$ such that (i) if s, t is in $E \times E$ then $K(s, t)$ is a linear transformation from S_0 to S_0 and (ii) K has the property (2.1), then K is a function from $E \times E$ to $B\{S_0, Q_0\}$ and, for each s, t in $E \times E$, $K(s, t)^* = K(t, s)$.*

Proof. If t is in E then, for each x in S_0, we see from (2.1) that $Q_0(x, K[t, t]x) \geq 0$: thus $K(t, t)$ is Hermitian and so belongs to $B\{S_0, Q_0\}$. It follows now from (2.1) that if s, t is in $E \times E$ then, for each x, y in $S_0 \times S_0$, $Q_0(x, K[s, t]y)^* = Q_0(y, K[t, s]x)$ and

$$|Q_0(x, K[s, t]y)|^2 \leq Q_0(x, K[s, s]x)Q_0(y, K[t, t]y):$$

hence $|K(s, t)|^2 \leq |K(s, s)|\,|K(t, t)|$ and $K(s, t)^* = K(t, s)$.

Theorem 2.3 *If each of K and K' is a kernel in the inner product space $\{S, Q\}$ of functions from E to $\{S_0, Q_0\}$, then $K' = K$.*

Proof. For each s, t in $E \times E$ and each x, y in $S_0 \times S_0$ we have

$$Q_0(K'[t, s]y, x) = Q(K'[\cdot, s]y, K[\cdot, t]x) = Q_0(y, K[s, t]x),$$

so that $K'(t, s) = K(s, t)^* = K(t, s)$, and the theorem is proved.

Theorem 2.4 *If K is a function from $E \times E$ to $B\{S_0, Q_0\}$ and K has the property (2.1), then an inner product space $\{S', Q'\}$ of functions from E to $\{S_0, Q_0\}$ in which K is the kernel is obtained as follows: S' consists of all functions of the form $\sum K(\cdot, t_j)x_j$ for some finite sequence $\{t_i, x_i\}$ in*

$E \times S_0$, and Q' is defined by

$$(2.2)\quad Q'(f, g) = \sum_{i,j} Q_0(x_i, K[t_i, u_j]y_j),$$

where $f(.) = \sum K(., t_i)x_i$ *and* $g(.) = \sum K(., u_j)y_j$.

PROOF. Clearly S' is a linear space. Moreover, if $f(.) = \sum K(., t_j)x_j$ and $g(.) = \sum K(., u_j)y_j = \sum K(., v_j)z_j$ then

$$\sum_{i,j} Q_0(x_i, K[t_i, u_j]y_j) = \sum_i Q_0(x_i, g[t_i]) = \sum_{i,j} Q_0(x_i, K[t_i, v_j]z_j);$$

it follows that (2.2) does define a function Q' from $S' \times S'$ to the numbers and, by (2.1), that Q' is an inner product for S' provided it is true that if $Q'(f, f) = 0$ then $f = 0$. If t, x is in $E \times S_0$ and f is in S' and $f(.) = \sum K(., t_j)x_j$ and $k(.) = K(., t)x$, then

$$(2.3)\quad Q'(f, k) = \sum_i Q_0(x_i, K[t_i, t]x) = Q_0(f[t], x),$$

and from this it follows that

$$(2.4)\quad |\, Q_0(f[t], x)\,|^2 \le Q'(f, f)Q'(k, k) = Q'(f, f)Q_0(x, K[t, t]x).$$

Now (2.4) shows that $Q'(f, f) = 0$ implies $f = 0$, so that $\{S', Q'\}$ is an inner product space. Since (2.3) shows that K is the kernel in $\{S', Q'\}$, the theorem is proved.

THEOREM 2.5 *If K is a function from $E \times E$ to $B\{S_0, Q_0\}$ and K has the property* (2.1) *then the only complete inner product space of functions from E to $\{S_0, Q_0\}$ in which K is the kernel in the space $\{S, Q\}$ obtained from $\{S', Q'\}$ (of Theorem* 2.4) *as follows:* (i) *S is the set of all functions f from E to S_0 such that there is a convergent sequence $\{f_n\}$ in $\{S', Q'\}$ such that, for each t in E, $\{f_n(t)\}$ has the limit $f(t)$ in $\{S_0, Q_0\}$, and* (ii) *if f, g is in $S \times S$ then*

$$(2.5)\quad Q(f, g) = \text{Limit } Q'(f_n, g_n) \text{ as } n \to \infty,$$

where $\{f_n\}$, $\{g_n\}$ is any ordered pair of convergent sequences in $\{S', Q'\}$ such that (for each t in E) $\{f_n(t)\}$ has the limit $f(t)$ and $\{g_n(t)\}$ has the limit $g(t)$ in $\{S_0, Q_0\}$.

PROOF. The usual *sequential completion* $\{S'', Q''\}$ of the space $\{S', Q'\}$ may be defined as follows: let T be a function from the class C of convergent sequences in $\{S', Q'\}$ such that if $\{f_n\}$ is in C then $T\{f_n\}$ is the subcollection of C to which $\{g_n\}$ belongs only in case $\{f_n - g_n\}$ has the limit 0 in $\{S', Q'\}$; then S'' is the image of C under the transformation T and, for each $\{f_n\}$, $\{g_n\}$ in $C \times C$ and each number z, one defines

$$(2.6)\quad \begin{cases} T\{f_n\} + T\{g_n\} = T\{f_n + g_n\}, \\ zT\{f_n\} = T\{zf_n\}, \\ \text{and}\quad Q''(T\{f_n\}, T\{g_n\}) \\ \qquad = \text{Limit } Q'(f_i, g_i) \quad \text{as } i \to \infty. \end{cases}$$

It follows from (2.4) that, for each f in S' and each t in E,

$$(2.7)\quad Q_0(f[t], f[t]) \le Q'(f, f)\,|\,K(t, t)\,|.$$

By the completeness of $\{S_0, Q_0\}$, there is a function T' from S'' such that if $\{f_n\}$ is in C then $T'T\{f_n\}$ is a function f from E to S_0 such that, for each t in E, $\{f_n(t)\}$ has the limit $f(t)$ in $\{S_0, Q_0\}$: the space S is the image of S'' under the linear transformation T'.

Suppose that T' is not reversible. Then there exists an element $\{f_n\}$ of C such that $T'T\{f_n\} = 0$ in S and $T\{f_n\} \neq 0$ in S''. From (2.3) we see that $\{Q'(f_n, g)\}$ has the limit 0 for each g in S'. Let M be a positive number such that $Q'(f_n, f_n) < M$ for $n = 1, 2, \cdots$, and let c be a positive number; there is a positive integer N such that $Q'(f_i - f_j, f_i - f_j) < (c/M)^2$ for all i and j greater than N; if i is an integer greater than N, then there is an integer j greater than N such that $|\,Q'(f_j, f_i)\,| < c$ so that

$$Q'(f_i, f_i) \le |\,Q'(f_i - f_j, f_i)\,| + |\,Q'(f_j, f_i)\,| < 2c.$$

But this implies that $T\{f_n\} = 0$ in S''—a contradiction. Therefore the formula,

$$Q(T'T\{f_n\}, T'T\{g_n\}) = Q''(T\{f_n\}, T\{g_n\}),$$

defines an inner product Q for S: $\{S, Q\}$ is complete since T' is an isomorphism from $\{S'', Q''\}$ onto $\{S, Q\}$, and it is readily seen that K is the kernel in $\{S, Q\}$.

Suppose that $\{S_1, Q_1\}$ is a complete inner product space of functions from E to $\{S_0, Q_0\}$ in which K is the kernel, and $\{S_1, Q_1\}$ is not $\{S, Q\}$. Since S_1 must contain S' and S' is dense in $\{S, Q\}$, then $\{S_1, Q_1\}$ is a proper extension of $\{S, Q\}$. Therefore there exists a function f in S_1 such that $f \neq 0$ and f is orthogonal to S in $\{S_1, Q_1\}$: but this implies that

$$Q_0(f[t], x) = Q_1(f[.], K[., t]x) = 0$$

for each t in E and each x in S_0, so that $f = 0$—a contradiction. The theorem is proved.

THEOREM 2.6 *If $\{S, Q\}$ is a complete inner product space of functions from E to $\{S_0, Q_0\}$, the following statements are equivalent:*

(i) *There is a kernel in $\{S, Q\}$.*

(ii) *For each t, x in $E \times S_0$ the equation $Lf = Q_0(f[t], x)$ defines a continuous linear function L from $\{S, Q\}$ to the numbers.*

PROOF. That (i) implies (ii) follows directly from the definition of a kernel. Suppose that (ii) is true. Then there is only one function g from $E \times E \times S_0$ to S such that

$$Q_0(f[t], x) = Q(f[\,.\,], g[\,.\,, t, x]) \tag{2.8}$$

for each f in S and each t, x in $E \times S_0$. It follows that

$$g(s, t, ax + by) = ag(s, t, x) + bg(s, t, y)$$

for each s, t in $E \times E$ and each x, y in $S_0 \times S_0$ and each ordered number pair a, b: hence there is a function K from $E \times E$ such that if s, t is in $E \times E$ then $K(s, t)$ is a linear transformation from S_0 to S_0 and, for each x in S_0, $K(s, t)x = g(s, t, x)$. By (2.8), K is the kernel in $\{S, Q\}$.

Remark. Theorem 2.6 suggests many examples of the type of space considered here: *e.g.*, suppose S_0 is not finite dimensional, T_0 is a linear transformation from the subset D of S_0 into S_0 which is closed and discontinuous with respect to Q_0, and E is the set of which the elements are the numbers 1 and 2; let S be the linear space of functions f from E such that $f(1)$ is in D and $f(2)$ is $T_0 f(1)$, and for f, g in $S \times S$ define $Q(f, g) = Q_0(f[1], g[1]) + Q_0(f[2], g[2])$.

THEOREM 2.7 *Suppose K is the kernel in the complete inner product space $\{S, Q\}$ of functions from E to $\{S_0, Q_0\}$. A necessary and sufficient condition that the function f, from E to S_0, belong to S is that there exist a non-negative number N such that*

$$|\sum Q_0(f[t_i], x_i)|^2 \leq N \sum_{i,j} Q_0(x_i, K[t_i, t_j]x_j) \tag{2.9}$$

for each finite sequence $\{t_i, x_i\}$ in $E \times S_0$; if f is in S then $Q(f, f)$ is the least N such that (2.9) holds.

PROOF. If f belongs to S then the indicated sum on the left in (2.9) is $Q(f, k)$ where $k(\,.\,) = \sum K(\,.\,, t_j)x_j$; since the set S' of such functions k (by Theorem 2.5) is dense in $\{S, Q\}$, then $Q(f, f)$ is the least N such that (2.9) holds. Suppose, now, that f is a function from E to S_0 such that (2.9) holds for some non-negative number N. It is readily seen that

$$Lg = \sum Q_0(x_i, f[t_i]) \quad \text{for } g(\,.\,) = \sum K(\,.\,, t_j)x_j \tag{2.10}$$

defines a continuous linear function L from the subset S' of $\{S, Q\}$ to the numbers. Since S' is dense in $\{S, Q\}$, there is a function f_0 in S such that for each g in S' $Lg = Q(g, f_0)$. Hence, for each t, x in $E \times S_0$, we have

$$Q_0(x, f[t]) = Q(K[\,.\,, t]x, f_0[\,.\,]) = Q_0(x, f_0[t]).$$

It follows that $f_0 = f$, and the theorem is proved.

THEOREM 2.8 *Under the supposition of Theorem 2.7, let G be a maximal orthonormal set in $\{S, Q\}$. For each s, t, x in $E \times E \times S_0$,*

$$K(s, t)x = \sum_g Q_0(x, g[t])g[s], \tag{2.11}$$

where the convergence (with respect to s) is in $\{S, Q\}$ and the summation is taken over any countable subset G' of G such that if g is in G and $Q_0(x, g[t]) \neq 0$ then g is in G'.

Proof is omitted since, by the defining property of K, (2.11) is merely the expansion with respect to G of the function k defined by $k(s) = K(s, t)x$. It should be noted that, in general, the set G' depends upon x and t.

THEOREM 2.9 *Under the supposition of Theorem 2.7, suppose further that $\{S_0, Q_0\}$ is separable and that E is a separable topological space. If the functions in S are weakly continuous—in the sense that if f is in S and x is in S_0 then $Q_0(f[\,.\,], x)$ is a continuous function from E to the numbers—then $\{S, Q\}$ is separable.*

PROOF. Let S_0' be a countable dense set in $\{S_0, Q_0\}$ and E' be a countable dense set in E. Let G be a maximal orthonormal set in $\{S, Q\}$ and G' be the subset of G to which g belongs only in case $Q_0(x, g[t]) \neq 0$ for some t, x in $E' \times S_0'$. Then G' is countable. Suppose G' is not G, so that there is a function f in G which is orthogonal to G'. By Theorem 2.8 and the defining property of K, $Q_0(f[t], x) = 0$ for all t, x in $E' \times S_0'$. Since S_0' is dense in $\{S_0, Q_0\}$, then $Q_0(f[t], x) = 0$ for all t, x in $E' \times S_0$. Finally, since E' is dense in E and f is weakly continuous, $Q_0(f[t], x) = 0$ for all t, x in $E \times S_0$, which implies that $f = 0$ in S—a contradiction. Hence G' is G, and $\{S, Q\}$ is separable.

Remark. Let $\{S, Q\}$ be the complete inner product space in which the function K, defined in Theorem 1.6, is the kernel. The condition

(Theorem 2.7) for the function f from $[0, 1]$ to S_0 to belong to S becomes: there is a non-negative number N such that

$$(2.12)\quad \begin{cases} |Q_0(f[0], y_0) + \sum_1^n Q_0(f[t_i] - f[t_{i-1}], y_i)|^2 \\ \quad \le N\,\{Q_0(y_0, Hy_0) \\ \qquad + \sum_1^n Q_0(y_i, [\phi(t_i) - \phi(t_{i-1})]y_i)\} \end{cases}$$

for each subdivision $\{t_i\}_0^n$ of $[0, 1]$ and each sequence $\{y_i\}_0^n$ in S_0. As an application of Theorem 2.9, it can be shown that if $\{S_0, Q_0\}$ is separable then $\{S, Q\}$ is separable: (1) using Lemma 1.3, one shows that there is a countable subset M of $[0, 1]$ such that if t, x is in $[0, 1] \times S_0$ then $K(\cdot, t)x$ is continuous on $[0, 1] - M$; (2) one defines the topological space E so that points of E are numbers of the interval $[0, 1]$, a neighborhood of a point p of M is the set of which the only element is p, and a neighborhood of a point p of $[0, 1] - M$ is the intersection with $[0, 1]$ of a number segment containing p; (3) by the inequality $|K(t, t)| \le |H + \phi(1)|$ and Theorem 2.5, one sees that the functions in S are continuous from E to $\{S_0, Q_0\}$.

3. Kernel systems

DEFINITION 3.1 A *kernel system* is a sequence $\{K, E, S, Q\}$ such that K is the kernel in the complete inner product space $\{S, Q\}$ of functions from the set E to $\{S_0, Q_0\}$.

LEMMA 3.1 *Suppose H is a non-negative Hermitian element of $B\{S_0, Q_0\}$, J is any element of $B\{S_0, Q_0\}$ such that $J^*J = H$, M is the closure in $\{S_0, Q_0\}$ of the linear manifold $J(S_0)$, and $S_1 = J^*(M)$. The following statements are true:*

(i) *The equation,*

$$Q_1(J^*x, J^*y) = Q_0(x, y) \text{ for } x, y \text{ in } M \times M,$$

defines an inner product Q_1 for S_1; the space $\{S_1, Q_1\}$ is complete and, if Q is the contraction of Q_0 to $M \times M$ and T is the contraction of J^* to M; T is an isomorphism from the space $\{M, Q\}$ onto $\{S_1, Q_1\}$.

(ii) *The point z of S_0 belongs to S_1 only in case there is a sequence $\{x_n\}$ in S_0 such that $Q_0(x_i - x_j, H[x_i - x_j]) \to 0$ and also*

$$Q_0(z - Hx_i, z - Hx_i) \to 0 \text{ as } i, j \to \infty,$$

in which case $Q_1(z, z)$ is the limit of the sequence $\{Q_0(x_n, Hx_n)\}$.

(iii) *The point z of S_0 belongs to S_1 only in case there is a non-negative number N such that $|Q_0(z, x)|^2 \le NQ_0(x, Hx)$ for all x in S, in which case $Q_1(z, z)$ is the least such N.*

No detailed proof will be given. To see that, in (i), T is reversible, one observes that M is the orthogonal complement in $\{S_0, Q_0\}$ of the set of all x such that $J^*x = 0$. The remaining assertions of the Lemma are direct applications of Theorems 2.4, 2.5, and 2.7 in connection with the kernel system $\{K_2, E_2, S_2, Q_2\}$, where E_2 is an arbitrary set and $K_2(s, t) = H$ for all s, t in $E_2 \times E_2$.

LEMMA 3.2 *Under the suppositions of Lemma 3.1, each of the following is a necessary and sufficient condition that S_1 be a closed linear manifold in $\{S_0, Q_0\}$:*

(i) $H(S_0) = S_1$. (ii) $J(S_0)$ *is closed.* (iii) $H(S_0)$ *is closed.*

PROOF. Suppose (i) is true. Then, since $J^*(M) = J^*(S_0)$, we have $J^*J(S_0) = J^*(S_0)$. If $\{x_n\}$ is a sequence in S_0 such that $\{Jx_n\}$ has the limit y in $\{S_0, Q_0\}$ then, since J^* is continuous, $\{J^*Jx_n\}$ has the limit J^*y. By supposition, there is a point z of S_0 such that $J^*y = J^*Jz$. Since $y - Jz$ is in M, this means that $y = Jz$. Thus we see that (i) implies (ii).

Suppose, now, that (ii) is true. Let M^* be the closure in $\{S_0, Q_0\}$ of the linear manifold $J^*(S_0)$. Then $J(M^*) = J(S_0)$, so that the contraction J_1 of J to M^* is reversibly continuous. Since M^* is the closure of $H(S_0)$, we see that $H(S_0) = H(M^*)$. Finally, since

$$|Q_0(J_1x, J_1x)|^2 \le Q_0(x, x)Q_0(Hx, Hx)$$

for x in M^*, we see that the contraction of H to M^* is reversibly continuous, and thence that $H(S_0)$ is closed. Therefore (ii) implies (iii).

If we suppose that (iii) is true then, since $H(S_0)$ is a dense subset of S_1, we see that (i) is true and that S_1 is closed.

Suppose, finally, that S_1 is closed. This means that $J^*(S_0)$ is closed. By applying the preceding arguments to $H_1 = JJ^*$, we see that $H_1(S_0)$ is closed and is $J(S_0)$, so that (ii) is true. This completes our proof.

THEOREM 3.1 *Suppose each of $\{K_1, E_1, S_1, Q_0\}$ and $\{K_2, E_2, S_2, Q_2\}$ is a kernel system. There exists a collection A_{12} of ordered triples such that all the following statements are true:*

(i) *The set of first terms of triples in A_{12} is the collection of continuous linear transformations from $\{S_1, Q_1\}$ to $\{S_2, Q_2\}$.*

(ii) *The set of second terms of triples in A_{12} is the collection of functions L from $E_1 \times E_2$ to $B\{S_0, Q_0\}$*

such that there exists a non-negative number N such that

$$|\sum_{i,j} Q_0(x_i, L[s_i, t_j]y_j)|^2 \leq$$

$$N\sum_{i,j} Q_0(x_i, K_1[s_i, s_j]x_j) \quad \sum_{i,j} Q_0(y_i, K_2[t_i, t_j]y_j)$$

for each finite sequence $\{x_i, s_i, t_i, y_i\}$ in $S_0 \times E_1 \times E_2 \times S_0$.

(iii) *The set of third terms of triples in A_{12} is the collection of continuous linear transformations from $\{S_2, Q_2\}$ to $\{S_1, Q_1\}$.*

(iv) *Each term of any triple in A_{12} determines the other two, in accordance with the following identities for $\{T_{21}, L, T_{12}\}$ in A_{12}:*

$$(3.1)\quad \begin{cases} Q_0(T_{21}f[t], y) = Q_1(f[\,.\,], L[\,.\,, t]y) \\ Q_2(L[s, \,.\,]^*x, g[\,.\,]) = Q_0(x, T_{12}g[s]) \\ Q_1(f, T_{12}g) = Q_2(T_{21}f, g) \end{cases}$$

for all f, g in $S_1 \times S_2$ and all x, s, t, y in $S_0 \times E_1 \times E_2 \times S_0$.

(v) *If $\{T_{21}, L, T_{12}\}$ is in A_{12} then $|T_{21}| = |T_{12}|$ and $|T_{21}|^2$ is the least N such that the inequality in* (ii) *holds as stated.*

We omit our proof, since this theorem is a direct application of ideas from Theorems 2.5, 2.6, and 2.7, and from our proofs of those theorems. It should be noted that $|T_{12}|$ in (v) is defined to be

$$\text{L.U.B. } Q_1(T_{12}g, T_{12}g)^{1/2}$$

for all g in S_2 such that $Q_2(g, g) = 1$, .

and that this is the same as L.U.B. $|Q_1(f, T_{12}g)|$ for all f, g in $S_1 \times S_2$ such that $Q_1(f, f) = Q_2(g, g) = 1$.

DEFINITION 3.2 If L is a function from $E_1 \times E_2$ to $B\{S_0, Q_0\}$ then L^* is the function from E_2 to E_1 to $B\{S_0, Q_0\}$ which is defined by the equation: $L^*(t, s) = L(s, t)^*$.

Theorem 3.1 has three direct corollaries which we now state without proof.

COROLLARY 3.1a *If the triple $\{T_{21}, L, T_{12}\}$ belongs to the class A_{12} (of Theorem* 3.1*) then the triple $\{T_{12}, L^*, T_{21}\}$ belongs to the corresponding class A_{21}, moreover $L(\,.\,, t)y = T_{12}K_2(\,.\,, t)y$ for each t, y in $E_2 \times S_0$—in the sense that if $g(v) = K_2(v, t)y$ for v in E_2 and $f = T_{12}g$ then f is in S_1 and $f(u) = L(u, t)y$ for u in E_1.*

COROLLARY 3.1b *If $\{T_{21}, L_1, T_{12}\}$ and $\{T_{32}, L_2, T_{23}\}$ belong respectively to the classes A_{12} and A_{23} (in the notation of Theorem* 3.1*) then $\{T_{32}T_{21}, L, T_{12}T_{23}\}$ belongs to the class A_{13}, where $L = L_1 \circ L_2$—in the sense that*

$$(3.2)\quad Q_0(x, L[s, t]y) = Q_2(L_1[s, \,.\,]^*x, L_2[\,.\,, t]y)$$

for each x, y in $S_0 \times S_0$ and each s, t in $E_1 \times E_3$.

COROLLARY 3.1c *Suppose $\{K, E, S, Q\}$ is a kernel system. Let $B(K)$ denote the linear space of functions L from $E \times E$ to $B\{S_0, Q_0\}$ satisfying the condition* (ii) *of Theorem* 3.1 *with $K_1 = K_2 = K$, and for L in $B(K)$ define $|L|$ to be the non-negative square root of the least N such that the inequality in* (ii) *holds. Then the equation,*

$$(3.3)\quad L(\,.\,, t)x = TK(\,.\,, t)x \text{ for } t, x \text{ in } E \times S_0,$$

defines a one-to-one correspondence $T \frown L$ between $B\{S, Q\}$ and $B(K)$ such that (i) *if $T \frown L$ then $T^* \frown L^*$ and $|T| = |L|$, and* (ii) *if $T_1 \frown L_1$ and $T_2 \frown L_2$ then $T_1 + T_2 \frown L_1 + L_2$ and $T_1T_2 \frown L$, where $L = L_1 \circ L_2$ (as in Corollary* 3.1b *with $Q_2 = Q$).*

Remark. Since the formal structure of our kernel systems seems much like that for the case that S_0 is a one-dimensional space (*i.e.*, the complex plane), it seems natural to ask whether or not—in the setting of the preceding Corollary—the class $B\{S, Q\}$ contains as "constant multipliers" all the elements of $B\{S_0, Q_0\}$. As is readily seen from our Illustration at the end of Chapter 1, the answer may be negative even in the case that S_0 is two-dimensional.

THEOREM 3.2 *Suppose each of $\{K_1, E, S_1, Q_1\}$ and $\{K_2, E, S_2, Q_2\}$ is a kernel system. A necessary and sufficient condition for S_1 to be a subset of S_2 is that there should exist a non-negative number N such that*

$$(3.4)\quad \sum_{i,j} Q_0(x_i, K_1[t_i, t_j]x_j) \leq N\sum_{i,j} Q_0(x_i, K_2[t_i, t_j]x_j)$$

for each finite sequence $\{t_i, x_i\}$ in $E \times S_0$; if S_1 is a subset of S_2 then the following statements are true:

(i) *There is a continuous linear transformation T_{12} from $\{S_2, Q_2\}$ to $\{S_1, Q_1\}$ such that $Q_1(f, T_{12}g) = Q_2(f, g)$ for all f, g in $S_1 \times S_2$.*

(ii) *If T_{21} is the transformation from $\{S_1, Q_1\}$ to $\{S_2, Q_2\}$ defined by the equation $T_{21}f = f$ for f in S_1, then $\{T_{21}, K_1, T_{12}\}$ belongs to the class A_{12} (of Theorem* 3.1*), and $|T_{12}|^2$ is the least N such that the condition* (3.4) *holds.*

(iii) *The contraction T_1 of T_{12} to S_1 (considered as a transformation in $\{S_1, Q_1\}$) is a non-negative*

Hermitian element of $B\{S_1, Q_1\}$ with norm $|T_1| = |T_{12}|^2$, and $Q_2(f, f) \leq |T_1| Q_1(f, f)$ for f in S_1.

PROOF. If there is an N such that (3.4) holds, then it follows from Theorems 2.5 and 2.7 that S_1 is a subset of S_2. Suppose, now, that S_1 is a subset of S_2. If f is in S_1 then we have

$$Q_1(f[\cdot], K_1[\cdot, t]x) = Q_0(f[t], x) = Q_2(f[\cdot], K_2[\cdot, t]x) \tag{3.5}$$

for each t, x in $E \times S_0$. That the equation $Q_1(f, T_{12}g) = Q_2(f, g)$ for f, g in $S_1 \times S_2$ defines a linear transformation T_{12} from S_2 to S_1 is now deduced from the following two considerations: (a) the linear manifold S_2' (as in Theorem 2.5) is dense in $\{S_2, Q_2\}$, and (b) if $\{h_n\}$ is a sequence in S_1 such that $\{Q_1(f, h_n)\}$ converges for each f in S_1 then there is an element h of S_1 such that, for each f in S_1, $\{Q_1(f, h_n)\}$ has the limit $Q_1(f, h)$. That T_{12} is closed follows from its defining equation and, since $\{S_2, Q_2\}$ is complete, it follows that T_{12} is continuous from $\{S_2, Q_2\}$ to $\{S_1, Q_1\}$. Thus statement (i) of the Theorem is established, and (ii) is an application of Theorem 3.1 with the observation that $K_1^* = K_1$. Concerning (iii), since we have $Q_1(f, T_1 f) = Q_2(f, f)$ for all f in S_1, we see that T_1 is Hermitian; hence T_1 is closed and therefore continuous in $\{S_1, Q_1\}$. A short computation shows that if N is a non-negative number then, in order that (3.4) hold, it is necessary and sufficient that for each f in S_1 $Q_2(f, f) \leq NQ_1(f, f)$. The remaining assertions follow immediately, and the proof is complete.

Remark 1. In the preceding Theorem, we see that $S_1 = S_2$ only in case K_1 belongs to $B(K_2)$ and K_2 belongs to $B(K_1)$ (as defined in Corollary 3.1c), *i.e.*, only in case there exists an ordered number pair N_1, N_2 such that (3.4) holds as stated with $N = N_1$, and holds with K_1 and K_2 interchanged and $N = N_2$; in this case T_{12} is reversible, $\{T_{21}^{-1}, K_2, T_{12}^{-1}\}$ belongs to the class A_{21}, and $|T_{12}^{-1}|^2$ is the least N_2 which can be used.

Remark 2. It should be noted that, for a given kernel system $\{K, E, S, Q\}$, we have a characterization of all possible systems of the form $\{K_1, E, S, Q_1\}$, namely: there must be a positive Hermitian element T_1 of $B\{S, Q\}$ such that $T_1 \backsim K_1$ as in Corollary 3.1c and such that T_1^{-1} belongs to $B\{S, Q\}$, and in this case

$$Q_1(f, T_1 g) = Q(f, g) \text{ and } Q_1(f, g) = Q(f, T_1^{-1} g) \tag{3.6}$$

for all f, g in $S \times S$.

Remark 3. If $\{K, E, S, Q\}$ is a kernel system and S_1 is a linear subspace of S, it is seen from Theorems 2.6 and 3.2 that, in order that there exist a function K_1 and a function Q_1 such that $\{K_1, E, S_1, Q_1\}$ is a kernel system, it is necessary and sufficient that there exist an inner product Q_3 for S_1 and a positive number N such that $Q(f, f) \leq NQ_3(f, f)$ for each f in S_1 and such that $\{S_1, Q_3\}$ is a complete inner product space.

The following Theorem is a direct application of Lemma 3.2 (with $\{S_0, Q_0\}$ replaced by $\{S_2, Q_2\}$), and no proof will be given.

THEOREM 3.3 *Suppose, with the notation of Theorem 3.2, that S_1 is a subset of S_2. Then T_{12}, regarded as an element of $B\{S_2, Q_2\}$, is a non-negative Hermitian transformation; moreover, if J is any element of $B\{S_2, Q_2\}$ such that $J^*J = T_{12}$ and M is the closure in $\{S_2, Q_2\}$ of linear manifold $J(S_2)$, then*

(i) *S_1 is $J^*(M)$.*

(ii) *If Q is the contraction of Q_2 to $M \times M$ and T is the contraction of J^* to M, then T is an isomorphism from the space $\{M, Q\}$ onto $\{S_1, Q_1\}$.*

(iii) *If either S_1 or $J(S_2)$ or $T_{12}(S_2)$ is a closed linear manifold in $\{S_2, Q_2\}$ then $S_1 = T_{12}(S_2)$; conversely, if $S_1 = T_{12}(S_2)$ then each of S_1 and $J(S_2)$ is a closed linear manifold in $\{S_2, Q_2\}$.*

THEOREM 3.4 *Suppose $\{K_1, E_1, S_1, Q_1\}$ and $\{K_2, E_2, S_2, Q_2\}$ are kernel systems such that E_1 is a proper subset of E_2 and K_1 is the contraction of K_2 to $E_1 \times E_1$, and let L be defined by the equation $L(s, t) = K_2(s, t)$ for s, t in $E_1 \times E_2$. Then L satisfies the condition* (ii) *of Theorem* 3.1 *with $N = 1$, and the element $\{T_{21}, L, T_{12}\}$ of the class A_{12} has the following properties:*

(i) *The transformation $T_{12}T_{21}$ is the identity in $B\{S_1, Q_1\}$.*

(ii) *The space S_1 is the image of S_2 under T_{12}, i.e., is the set of contractions to E_1 of functions in S_2.*

(iii) *The transformation $T_{21}T_{12}$ is a projection in $B\{S_2, Q_2\}$, with image set S_3 such that the orthogonal complement of S_3 in $\{S_2, Q_2\}$ is the set of all f in S_2 such that $f(t) = 0$ for all t in E_1.*

(iv) *If Q_3 is the contraction of Q_2 to $S_3 \times S_3$, then T_{21} is an isomorphism from the space $\{S_1, Q_1\}$ onto $\{S_3, Q_3\}$, and its inverse is the contraction of T_{12} to S_3.*

(v) *If f is in S_2 then $Q_1(T_{12}f, T_{12}f) \leq Q_2(f, f)$, where "=" holds only in case f belongs to S_3.*

PROOF. By Corollaries 3.1b and 3.1c, $T_{12}T_{21}$ is in $B\{S_1, Q_1\}$ and corresponds to the element $M = L \circ L^*$ of $B(K_1)$, where

$$Q_0(x, M[s, t]y) = Q_2(L[s, .]^*x, L^*[., t]y)$$
$$= Q_2(K_2[., s]x, K_2[., t]y) = Q_0(x, K_2[s, t]y)$$
$$= Q_0(x, K_1[s, t]y)$$

for each x, y in $S_0 \times S_0$ and each s, t in $E_1 \times E_1$: thus M is K_1 and (i) is established. From the second of the identities (3.1) we see that if g is in S_2 and $f = T_{12}g$ then for each s, x in $E_1 \times S_0$

$$Q_0(x, f[s]) = Q_2(K_2[., s]x, g[.]) = Q_0(x, g[s]),$$

and this together with (i) establishes statement (ii). Let T denote the element $T_{21}T_{12}$ of $B\{S_2, Q_2\}$. It follows from (i) that $T_{12}T = T_{12}$ and, by the third of the identities (3.1),

$$Q_1(T_{12}f, T_{12}g) = Q_2(Tf, g) \quad \text{for all } f, g \text{ in } S_2 \times S_2, \tag{3.7}$$

so that T is Hermitian; moreover, for f, g in $S_2 \times S_2$, we have

$$Q_2(Tf, Tg) = Q_1(T_{12}f, T_{12}Tg) = Q_1(T_{12}f, T_{12}g)$$
$$= Q_2(Tf, g),$$

so that $T^2 = T$: thus T is a projection. From (3.7), $Tf = 0$ in S_2 only in case $T_{12}f = 0$ in S_1, and this together with (ii) establishes statement (iii). Assertions (iv) and (v) follow now from (3.7) and the consequent identity,

$$Q_2(f, g) = Q_1(T_{12}f, T_{12}g) + Q_2([1 - T]f, [1 - T]g), \tag{3.8}$$

and this concludes our argument.

THEOREM 3.5 *Suppose $\{K, E, S, Q\}$ is a kernel system, $\{S_1, Q_1\}$ is an inner product space, and T is a continuous linear function from $\{S, Q\}$ to $\{S_1, Q_1\}$. Let T^* be the function defined by $Q_1(Tf, x) = Q(f, T^*x)$, so that TT^* is Hermitian in $B\{S_1, Q_1\}$. If M is the set of all f in S such that $Tf = 0$ in S_1, the following are true:*

(i) *If f is in S and Tf is in $TT^*(S_1)$,*

$$P_M f = f - T^*[TT^*]^{-1}Tf \tag{3.9}$$

where $[TT^*]^{-1}Tf$ is any x in S_1 such that $TT^*x = Tf$, and P_M is the projection in $B\{S, Q\}$ with image set M.

(ii) *If x is in $TT^*(S_1)$, $Q_1(x, [TT^*]^{-1}x)$ is the minimum value of $Q(f, f)$ for all f in S such that $Tf = x$—this minimum is attained only in case $P_M f = 0$ in S.*

THEOREM 3.6 *Under the suppositions of Theorem 3.5, if x is a point of S_1 such that $TT^*x \neq 0$ and M is the set of all f in S such that $Q_1(Tf, x) = 0$, the following are true:*

(i) *If f is in S then*

$$P_M f = f - \frac{Q_1(Tf, x)}{Q_1(TT^*x, x)} f \tag{3.10}$$

where P_M is the projection in $B\{S, Q\}$ with image set M.

(ii) *If b is a number, $|b|^2/Q_1(TT^*x, x)$ is the minimum value of $Q(f, f)$ for all f in S such that $Q_1(Tf, x) = b$—this minimum is attained only in case $P_M f = 0$ in S.*

Proof is omitted for the preceding two theorems—the assertions are results of straightforward computation.

Remark 1. If S_1 is one-dimensional, the preceding two theorems are equivalent.

Remark 2. By Corollary 3.1c, the formulas (3.9) and (3.10), respectively, define the element L of $B(K)$ to which P_M corresponds in each case.

Remark 3. To illustrate Remark 2, we consider the kernel system $\{K, [0, 1], S, Q\}$ where K is the function defined in Theorem 1.6—this is the system mentioned at the end of Chapter 2. In Theorem 3.5, letting $\{S_1, Q_1\}$ be $\{S_0, Q_0\}$, each of C and D be in $B\{S_0, Q_0\}$, and $Tf = Cf(0) + Df(1)$, we find TT^* given by

$$(C + D)H(C + D)^* + D\phi(1)D^*; \tag{3.11}$$

if, moreover, $P_M \sim L$ and t, y is in $[0, 1] \times S_0$ and the point $[(C + D)H + D\phi(t)]y$ belongs to $TT^*(S_0)$, then (3.9) gives us the following expression for $K(s, t)y - L(s, t)y$:

$$[(C + D)H + D\phi(s)]^*[TT^*]^{-1}[(C + D)H + D\phi(t)]y \tag{3.12}$$

and this is valid for all s in $[0, 1]$.

4. IDEAL KERNEL SYSTEMS

DEFINITION 4.1 The kernel system $\{K, E, S, Q\}$ is *ideal* provided that, for each finite sequence $\{t_i\}$ in E, the set of all functions of the form $\sum K(., t_j)x_j$ (with the x_j in S_0) is a closed linear manifold in $\{S, Q\}$.

DEFINITION 4.2 If n is a positive integer then S_0^n is the linear space to which x belongs only in case x is a sequence $\{x_i\}_1^n$ in S_0, and Q_0^n is the inner product for S_0^n which is defined by:

$$Q_0^n(x, y) = \sum_1^n Q_0(x_i, y_i) \quad \text{for } x = \{x_i\}_1^n \text{ and } y = \{y_i\}_1^n. \tag{4.1}$$

For any kernel system $\{K, E, S, Q\}$, if each of $\{s_i, x_i\}$ and $\{t_i, y_i\}$ is a finite sequence in $E \times S_0$ then

$$Q(\sum K[\cdot, s_i]x_i, \sum K[\cdot, t_j]y_j) = \sum_{i,j} Q_0(x_i, K[s_i, t_j]y_j); \tag{4.2}$$

thus we see that Theorem 2.5 together with Lemmas 3.1 and 3.2, the latter applied to spaces $\{S_0^n, Q_0^n\}$, give us two characterizations of ideal kernel systems—we state these as separate theorems and omit the proofs.

THEOREM 4.1 *Suppose $\{K, E, S, Q\}$ is a kernel system, n is a positive integer, $t = \{t_i\}_1^n$ is a sequence in E, and $M(t)$ is the linear manifold in S to which f belongs only in case there is a point $\{x_i\}_1^n$ in S_0^n such that $f(\cdot) = \sum K(\cdot, t_i)x_i$. A necessary and sufficient condition that $M(t)$ be closed in $\{S, Q\}$ is that, if H is the linear transformation from S_0^n to S_0^n defined by*

$$Hx = \{\sum K(t_i, t_j)x_j\}_1^n \quad \text{for} \quad x = \{x_i\}_1^n, \tag{4.3}$$

then $H(S_0^n)$ is a closed linear manifold in $\{S_0^n, Q_0^n\}$.

THEOREM 4.2 *Under the suppositions of Theorem 4.1, another necessary and sufficient condition that $M(t)$ be closed in $\{S, Q\}$ is that, if $V(t)$ is the subset of S_0^n to which x belongs only in case there is a function f in S such that $x = \{f(t_i)\}_1^n$, then $V(t)$ is a closed linear manifold in $\{S_0^n, Q_0^n\}$.*

Remark 1. If S_0 is finite dimensional, then every kernel system is ideal—since S_0^n is also finite dimensional for all n; on the other hand, if S_0 is not finite dimensional and E is any set, a kernel system which is not ideal is obtained by letting H be any non-negative Hermitian element of $B\{S_0, Q_0\}$ such that $H(S_0)$ is not closed in $\{S_0, Q_0\}$ and defining $K(s, t)$ to be H for all s, t in $E \times E$.

Remark 2. It follows from Theorem 4.2 that, if $\{K, E, S, Q\}$ is an ideal kernel system, then every system of the form $\{K_1, E, S, Q_1\}$ is also ideal.

THEOREM 4.3 *Under the suppositions of Theorem 4.1, another necessary and sufficient condition that $M(t)$ be closed in $\{S, Q\}$ is that, if f belongs to S, then there is a point $\{x_i\}_1^n$ of S_0^n such that $\{\sum K(t_i, t_j)x_j\}_1^n$ is $\{f(t_i)\}_1^n$; if $M(t)$ is closed and f is a function in S and $\{x_i\}_1^n$ is a point in S_0^n such that $\{\sum K(t_i, t_j)x_j\}_1^n$ is $\{f(t_i)\}_1^n$, then $P_t f(\cdot) = \sum K(\cdot, t_i)x_i$—where P_t is the projection in $B\{S, Q\}$ with image set $M(t)$.*

No detailed proof will be given. The equivalence of the condition here stated to the conditions given in Theorems 4.1 and 4.2 is another application of Lemma 3.2 in the space $\{S_0^n, Q_0^n\}$. The assertion about the projection P_t follows directly from the observation that if f belongs to S then f is orthogonal to $M(t)$ in $\{S, Q\}$ only in case $\{f(t_i)\}_1^n = 0$ in S_0^n.

THEOREM 4.4 *Suppose $\{K, E, S, Q\}$ is an ideal kernel system, and for each finite sequence $t = \{t_i\}$ in E let P_t be the projection in $B\{S, Q\}$ with image set the collection of all functions of the form $\sum K(\cdot, t_j)x_j$. Then Q is a* **Hellinger integral** *over E, in the following sense: if f, g is in $S \times S$ then $Q(f, g)$ is a number J such that for each positive number c there is a finite sequence s in E such that if t is a finite sequence in E of which s is a subsequence then $|J - Q(f, P_t g)| < c$.*

Proof is omitted since this is a direct application of Theorems 2.5 and 4.3.

Illustration. Suppose $\{S, Q\}$ is the inner product space described in Theorem 1.4, H and ϕ satisfy the conditions (i) and (ii) in Theorem 1.5, and K is the function described in Theorem 1.6. Then $\{K, [0, 1], S, Q\}$ is an ideal kernel system. If t is a subdivision of the interval $[0, 1]$ and f, g is in $S \times S$ then $Q(f, P_t g)$ is the corresponding *approximating sum* for the integral

$$Q(f, g) = Q_0(f[0], H^{-1}g[0]) + \int_0^1 Q_0(df, [d\phi]^{-1}dg). \tag{4.4}$$

As noted in connection with Theorem 1.1, approximating sums for the integral $Q(f, f)$ are non-decreasing with successive refinements of a subdivision of $[0, 1]$—this application of Lemma 1.1 may now be interpreted in terms of elementary properties of projections: if t is a subdivision of $[0, 1]$ and u is a refinement of t then, for each f in S, $Q(f, P_t f) \leq Q(f, P_u f)$. If $t = \{t_i\}_0^n$ is a subdivision of $[0, 1]$, $\{x_i\}_0^n$ is a sequence in S_0, and $\{y_i\}_0^n = \{\sum_i^n x_j\}_0^n$, then for every f in S such that

$$f(0) = Hy_0 \quad \text{and} \quad \{f(t_i) - f(t_{i-1})\}_1^n = \{[\phi(t_i) - \phi(t_{i-1})]y_i\}_1^n, \tag{4.5}$$

one finds that $P_t f$ is the function $\sum K(\cdot, t_j)x_j$.

5. A system of Stieltjes integral equations

We are concerned now with ordered triples f, g, h of functions of bounded variation from $[0, 1]$ to $\{S_0, Q_0\}$ such that

$$(5.1)\qquad \begin{cases} f(b) - f(a) + \int_a^b d\phi \cdot g = 0 & \text{and} \\ \int_a^b d\theta \cdot f + g(b) - g(a) = h(b) - h(a) \end{cases}$$

for each subinterval $[a, b]$ of $[0, 1]$,

where these are the mean integrals which we have previously studied in connection with continuous products and quasi-harmonic operators [10],[7] and where we postulate for ϕ and θ the following:

I. Each of ϕ and θ is a function from $[0, 1]$ to $B\{S_0, Q_0\}$ which is of bounded variation on $[0, 1]$.

II. $\phi(0) = \theta(0) = 0$ and, for each subinterval $[a, b]$ of $[0, 1]$, each of $\phi(b) - \phi(a)$ and $\theta(b) - \theta(a)$ is Hermitian and non-negative.

III. $\theta(1)$ is reversibly continuous and, for each subinterval $[a, b]$ of $[0, 1]$, $[\phi(b) - \phi(a)](S_0)$ is a closed linear manifold in $\{S_0, Q_0\}$.

Immediately before Theorem 5.4 we shall add to this list a fourth postulate—which will place a restriction on possible common discontinuities of ϕ and θ. We note that, if S_0 is finite dimensional, then II implies I.

It will be important to have one more variety of integral at our disposal, and a typical example of this variety is defined as follows:

DEFINITION 5.1 Suppose G is a function from $[0, 1]$ to $B\{S_0, Q_0\}$, each of f and h is a function from $[0, 1]$ to S_0, and for each subdivision $t = \{t_i\}_0^n$ of $[0, 1]$ the number

$$\sum_1^n \tfrac{1}{4} Q_0([f(t_i) + f(t_{i-1})], [G(t_i) - G(t_{i-1})][h(t_i) + h(t_{i-1})])$$

is denoted by $\sum_t Q_0(f, dG \cdot h)$: the integral $\int_0^1 Q_0(f, dG \cdot h)$ is the Hellinger-type limit at 0, 1 of $\sum Q_0(f, dG \cdot h)$, *i.e.*, is a number J such that for each positive number c there is a subdivision s of $[0, 1]$ such that $|J - \sum_t Q_0(f, dG \cdot h)| < c$ for each refinement t of s.

That the integral $\int_0^1 Q_0(f, dG \cdot h)$ exists if each of f and h is a quasi-continuous function from $[0, 1]$ to $\{S_0, Q_0\}$ and G is a function of bounded variation from $[0, 1]$ to $B\{S_0, Q_0\}$, as well as other elementary existence theorems for our various integrals, we shall not digress to prove. Proofs are easily constructed along the lines which we have previously pursued in connection with the theory of quasi-harmonic operators [10].

LEMMA 5.1 *Suppose G is a function of bounded variation from $[0, 1]$ to $B\{S_0, Q_0\}$, h is a quasi-continuous function from $[0, 1]$ to $\{S_0, Q_0\}$, and $g(u) = \int_0^u dG \cdot h$ for each u in $[0, 1]$. Then g is a function of bounded variation from $[0, 1]$ to $\{S_0, Q_0\}$ and the equation,*

$$(5.2)\qquad \int_0^1 Q_0(f, dg) = \int_0^1 Q_0(f, dG \cdot h),$$

holds for each quasi-continuous function f from $[0, 1]$ to $\{S_0, Q_0\}$.

PROOF. All is apparent except (5.2), and this follows directly from the inequality,

$$|\sum_t Q_0(f, dg) - \sum_t Q_0(f, dG \cdot h)| \leq \|f\|_{[0,1]} \sum_t \|dg - dG \cdot h\|,$$

by application of Theorem 3.3 of [10].

LEMMA 5.2 *Suppose h is a quasi-continuous function from $[0, 1]$ to $\{S_0, Q_0\}$ and $g(u) = \int_0^u d\phi \cdot h$ for each u in $[0, 1]$. Then the integral $\int_0^1 Q_0(dg, [d\phi]^{-1}dg)$ exists and*

$$(5.3)\qquad \int_0^1 Q_0(df, h) = \int_0^1 Q_0(df, [d\phi]^{-1}\, dg)$$

for each f from $[0, 1]$ to S_0 such that $\int_0^1 Q_0(df, [d\phi]^{-1}df)$ exists.

PROOF. By Theorems 3.2 and 3.3, it follows from postulates II and III that if $[a, b]$ is a subinterval of $[0, 1]$ then, for each subinterval $[c, d]$ of $[a, b]$, $[\phi(d) - \phi(c)](S_0)$ is a subset of $[\phi(b) - \phi(a)](S_0)$. Hence, for each subinterval $[a, b]$ of $[0, 1]$, $g(b) - g(a)$ is in $[\phi(b) - \phi(a)](S_0)$ and, by Lemma 1.1,

$$Q_0([\textstyle\sum_t d\phi \cdot h], [\phi(b) - \phi(a)]^{-1}[\sum_t d\phi \cdot h]) \leq \sum_t Q_0(h, d\phi \cdot h)$$

for each subdivision t of $[a, b]$; it follows that

$$Q_0([g(b) - g(a)], [\phi(b) - \phi(a)]^{-1}[g(b) - g(a)]) \leq \int_a^b Q_0(h, d\phi \cdot h).$$

Thus, by Theorem 1.2, $\int_0^1 Q_0(dg, [d\phi]^{-1}dg)$ exists and

$$\int_0^1 Q_0(dg, [d\phi]^{-1}dg) \leq \int_0^1 Q_0(h, d\phi \cdot h).$$

[7] The integral $\int_a^b d\phi \cdot g$ is the Hellinger-type limit in $\{S_0, Q_0\}$ of $\sum d\phi \cdot g$, where $\sum_t d\phi \cdot g = \frac{1}{2}\sum_1^n [\phi(t_i) - \phi(t_{i-1})][g(t_i) + g(t_{i-1})]$ for $t = \{t_i\}_0^n$ a monotone sequence with $t_0 = a$ and $t_n = b$.

Suppose now that $\int_0^1 Q_0(df, [d\phi]^{-1}df)$ exists, denote it by N, and let t be a subdivision of $[0, 1]$: by Schwarz's inequality and the last inequality above, it is readily found that

$$|\sum_t Q_0(df, [d\phi]^{-1}\, dg) - \sum_t Q_0(df, h)|^2 \leq N\left\{\int_0^1 Q_0(h, d\phi\cdot h) - 2\mathrm{Re}\sum_t Q_0(h, dg) + \sum_t Q_0(h, d\phi\cdot h)\right\}.$$

Our proof is completed by application of the preceding Lemma.

THEOREM 5.1 *Let S be the linear space of all functions f from $[0, 1]$ to $\{S_0, Q_0\}$ such that $\int_0^1 Q_0(df, [d\phi]^{-1}df)$ exists, Q_1 be the inner product for S defined by*

$$Q_1(f, g) = Q_0(f[0], g[0]) + \int_0^1 Q_0(df, [d\phi]^{-1}\, dg), \tag{5.4}$$

and K_1 be the function from $[0, 1] \times [0, 1]$ defined by $K_1(s, t) = K_1(t, s) = 1 + \phi(s)$ for $s \leq t$. Then:

(i) *$\{K_1, [0, 1], S, Q_1\}$ is an ideal kernel system.*

(ii) *if $\{S_0, Q_0\}$ is separable then $\{S, Q_1\}$ is separable.*

(iii) *a necessary and sufficient condition for the function K from $[0, 1]$ to $B\{S_0, Q_0\}$ to belong to $B(K_1)$—as in Corollary 3.1c—is that there exists a number N such that, if each of $\{s_i\}_0^m$ and $\{t_j\}_0^n$ is is a non-decreasing sequence in $[0, 1]$ and each of $\{x_i\}_0^m$ and $\{y_j\}_0^n$ is a sequence in S_0, then*

$$\left|Q_0(x_0, K[s_0, t_0]y_0) + \sum_1^m Q_0(x_i, [K(s_i, t_0) - K(s_{i-1}, t_0)]y_0) + \sum_1^m \sum_1^n Q_0(x_i, [K(s_i, t_j) - K(s_{i-1}, t_j) - K(s_i, t_{j-1}) + K(s_{i-1}, t_{j-1})]y_j) + \sum_1^n Q_0(x_0, [K(s_0, t_j) - K(s_0, t_{j-1})]y_j)\right|^2 \leq N\cdot\left\{Q_0(x_0, [1 + \phi(s_0)]x_0) + \sum_1^m Q_0(x_i, [\phi(s_i) - \phi(s_{i-1})]x_i)\right\}\cdot\left\{Q_0(y_0, [1 + \phi(t_0)]y_0) + \sum_1^n Q_0(y_j, [\phi(t_j) - \phi(t_{j-1})]y_j)\right\}.$$

THEOREM 5.2 *Let Q be the function from $S \times S$ defined by*

$$Q(f, g) = \int_0^1 Q_0(df, [d\phi]^{-1}\, dg) + \int_0^1 Q_0(f, d\theta\cdot g). \tag{5.5}$$

Then $\{S, Q\}$ is a complete inner product space in which there exists a kernel.

PROOF. It is clear that $\{S, Q\}$ is an inner product space, since $Q(f, f) = 0$ implies that f is constant and, by postulate III, this constant must be 0 in S_0. In showing that $\{S, Q\}$ is complete we will find that convergence of $\{f_n\}$ in $\{S, Q\}$ implies convergence of $\{f_n(r)\}$ in $\{S_0, Q_0\}$ uniformly for r in $[0, 1]$: by Theorem 2.6, this will establish the existence of a kernel in $\{S, Q\}$.

Suppose now that $\{f_n\}$ is a convergent sequence in $\{S, Q\}$. Let $\{g_n\}$ be the sequence in S defined by $g_n(s) = f_n(0) - f_n(s)$. Then certain facts become apparent in the following order (we omit details):

(a) $\{g_n\}$ is a convergent sequence in $\{S, Q_1\}$.

(b) there exists a function g from $[0, 1]$ to S_0 such that $\{g_n(r)\}$ has the limit $g(r)$ in $\{S_0, Q_0\}$ uniformly for r in $[0, 1]$.

(c) g belongs to S and $\{g_n\}$ has the limit g in $\{S, Q\}$.

(d) $\{f_n + g_n\}$ is convergent in $\{S, Q\}$, and $\{f_n(0)\}$ has a limit x in $\{S_0, Q_0\}$.

(e) $\{f_n(r)\}$ has the limit $f(r) = x - g(r)$ in $\{S_0, Q_0\}$ uniformly for r in $[0, 1]$.

(f) f belongs to S and $\{f_n\}$ has the limit f in $\{S, Q\}$.

This is the desired result, and completes our argument.

Remark. If K is the kernel in the space $\{S, Q\}$ then, as noted in Remark 1 following Theorem 3.2, K_1 belongs to $B(K)$ and K belongs to $B(K_1)$.

THEOREM 5.3 *Let K be the kernel in the space $\{S, Q\}$. Then the equations,*

$$\begin{cases} L'(s, t) = -\int_0^s d\theta(u)\cdot K(u, t) & \text{and} \\ L''(s, t) = \int_s^1 d\theta(u)\cdot K(u, t), \end{cases} \tag{5.6}$$

define functions L' and L'' from $[0, 1] \times [0, 1]$ to

$B\{S_0, Q_0\}$ *such that, for all* s *and* t *in* $[0, 1]$, $L''(s, t) - L'(s, t) = 1$ *and*

$$(5.7)\quad \begin{cases} K(s,t) - K(0,t) + \int_0^s d\phi(u)\cdot L'(u,t) \\ \qquad = 0 \text{ if } s \le t \quad \text{and} \\ K(1,t) - K(s,t) + \int_s^1 d\phi(u)\cdot L''(u,t) \\ \qquad = 0 \text{ if } s \le t. \end{cases}$$

PROOF. Since K belongs to $B(K_1)$ then it follows from Theorem 5.1 (iii) that for each t in $[0, 1]$ the function $K(\cdot, t)$ is quasi-continuous from $[0, 1]$ to $B\{S_0, Q_0\}$: thus we see that the system (5.6) does define functions L' and L'' from $[0, 1] \times [0, 1]$ to $B\{S_0, Q_0\}$. The remaining assertions follow from the computation,

$$\begin{aligned} Q_0(x, K_1[s,t]y) &= Q(K_1[\cdot, s]x, K[\cdot, t]y) \\ &= Q_1(K_1[\cdot, s]x, K[\cdot, t]y) - Q_0(K_1[0, s]x, K[0, t]y) \\ &\qquad + \int_0^1 Q_0(K_1[u, s]x, d\theta(u)\cdot K[u, t]y) \\ &= Q_0(x, K[s, t]y) - Q_0(x, K[0, t]y) \\ &\qquad + \int_0^1 Q_0(K_1[u, s]x, d\theta(u)\cdot K[u, t]y), \end{aligned}$$

and the consequent identity,

$$K_1(s,t) = K(s,t) - K(0,t) + \int_0^1 K_1(s,u)\cdot d\theta(u)\cdot K(u,t).$$

Remark. At this point we see, from Lemmas 5.1 and 5.2, that if f, g, h is a triple satisfying the system (5.1) then f belongs to S and the following computation is justified:

$$\begin{aligned} Q_0(f[s], x) &= Q(f[\cdot], K[\cdot, s]x) \\ &= \int_0^1 Q_0(df[t], [d\phi(t)]^{-1}\, dK[t, s]x) \\ &\qquad + \int_0^1 Q_0(f[t], d\theta(t)\cdot K[t, s]x) \\ &= -\int_0^1 Q_0(g[t], dK[t, s]x) \\ &\qquad - \int_0^1 Q_0(d\{g[t] - h[t]\}, K[t, s]x) \\ &= Q_0(g[0], K[0, s]x) - Q_0(g[1], K[1, s]x) \\ &\qquad + \int_0^1 Q_0(dh[t], K[t, s]x), \end{aligned}$$

and this shows that for each s in $[0, 1]$

$$(5.8)\quad f(s) = K(s,0)g(0) - K(s,1)g(1) + \int_0^1 K(s,t)\cdot dh(t).$$

In addition to I, II, and III, we now postulate for ϕ and θ the following:

IV. The function F, from $[0, 1]$ to $B\{S_0^2, Q_0^2\}$, which is defined by

$$F(r) = \begin{pmatrix} 0 & \phi(r) \\ \theta(r) & 0 \end{pmatrix},$$

satisfies the condition that $[F(r) - F(r-)]^2 = 0$ for each r in $(0, 1]$ and $[F(r+) - F(r)]^2 = 0$ for each r in $[0, 1)$.

This postulate IV allows us to treat the system (5.1) within the framework of our theory of continuous products and quasi-harmonic operators [10], and further to connect it with the kernel system $\{K, [0, 1], S, Q\}$. We let M be the function from $[0, 1] \times [0, 1]$ to $B\{S_0^2, Q_0^2\}$ defined by $M(s, t) = {}_s\prod^t \{I + dF\}$, where I denotes the identity in $B\{S_0^2, Q_0^2\}$. Now, M is equivalently defined by each of the equations,

$$M(s,t) = I + \int_s^t dF(u)\cdot M(u,t)$$

$$\text{and } M(s,t) = I + \int_s^t M(s,u)\cdot dF(u),$$

and has the property that $M(r, s)M(s, t) = M(r, t)$ for all r, s, and t in $[0, 1]$. From these considerations, one readily deduces that

$$M_{12}(s,t)^* = -M_{12}(t,s),$$

$$M_{21}(s,t)^* = -M_{21}(t,s), \text{ and}$$

$$M_{11}(s,t)^* = M_{22}(t,s)$$

for all s and t in $[0, 1]$. The function M has interesting connections with certain non-linear integral equations and with continuous continued fractions, even when S_0 is assumed merely to be a linear normed complete (or Banach) space: the reader is referred to [9] (esp. section 4) for the case that F is continuous, and to [10] (esp. section 5) for the more general case allowed by postulates I and IV.

THEOREM 5.4 *The transformation* $M_{21}(0, 1)$ *is reversibly continuous,* $M_{21}(0, 1)^{-1} = K(1, 0)$, *and*

for each s, t in $[0, 1] \times [0, 1]$

$$K(s,t) = \begin{cases} M_{11}(s,0)K(0,1)M_{22}(1,t) \\ \qquad\qquad \text{if } s \le t, \text{ and} \\ M_{11}(s,1)K(1,0)M_{22}(0,t) \\ \qquad\qquad \text{if } s \ge t. \end{cases} \tag{5.9}$$

PROOF. It follows from Theorem 5.3 and the properties of M that the matrix equations,

$$\begin{Bmatrix} K(s,t) \\ L'(s,t) \end{Bmatrix} = M(s,0) \begin{Bmatrix} K(0,t) \\ 0 \end{Bmatrix}$$

$$\text{and} \begin{Bmatrix} K(s,t) \\ L''(s,t) \end{Bmatrix} = M(s,1) \begin{Bmatrix} K(1,t) \\ 0 \end{Bmatrix},$$

must hold for $s \le t$ and $s \ge t$, respectively. From these and the identity $L''(t,t) - L'(t,t) = 1$ we then find that

$$M(t,1) \begin{Bmatrix} K(1,t) \\ 0 \end{Bmatrix} - M(t,0) \begin{Bmatrix} K(0,t) \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}.$$

Now, using the multiplicative properties of M, one obtains the two relations $M_{21}(0,1)K(1,t) = M_{22}(0,t)$ and $-M_{21}(1,0)K(0,t) = M_{22}(1,t)$, of which the latter means that $K(t,0)M_{21}(0,1) = M_{11}(t,1)$. Hence, taking $t = 0$ in the former and $t = 1$ in the latter, we have the equations $M_{21}(0,1)K(1,0) = K(1,0)M_{21}(0,1) = 1$. The remaining assertions of the theorem follow directly from the formulas which we have already indicated here. This concludes our argument.

We now present a theorem which extends the results indicated in the Remark following Theorem 5.3.

THEOREM 5.5 *If f, g, h is an ordered triple of functions of bounded variation from $[0, 1]$ to $\{S_0, Q_0\}$ then, in order that (5.1) hold, it is necessary and sufficient that f be given by (5.8) and*

$$\begin{aligned} g(s) &= L''(s,0)g(0) - L'(s,1)g(1) \\ &\quad + \int_0^s L''(s,t)\cdot dh(t) + \int_s^1 L'(s,t)\cdot dh(t) \end{aligned} \tag{5.10}$$

for each s in $[0, 1]$.

PROOF. Having already seen that if (5.1) holds then f is given by (5.8), we consider now the case that $g(0) = g(1) = 0$.

The only ordered pair f, g such that $f(0) = \int_0^1 K(0,t)\cdot dh(t)$ and $g(0) = 0$ and (5.1) holds (Theorem 4.4 of [10]) is the ordered pair f, g defined by

$$\begin{cases} f(s) = M_{11}(s,0)\int_0^1 K(0,t)\cdot dh(t) \\ \qquad\qquad + \int_0^s M_{12}(s,t)\cdot dh(t), \\ g(s) = M_{21}(s,0)\int_0^1 K(0,t)\cdot dh(t) \\ \qquad\qquad + \int_0^s M_{22}(s,t)\cdot dh(t). \end{cases}$$

This latter pair of equations is clearly equivalent to

$$\begin{cases} f(s) = \int_s^1 K(s,t)\cdot dh(t) + \int_0^s N_1(s,t)\cdot dh(t), \\ g(s) = \int_s^1 L'(s,t)\cdot dh(t) + \int_0^s N_2(s,t)\cdot dh(t), \end{cases}$$

where we have put

$$\begin{cases} N_1(s,t) = M_{11}(s,0)K(0,1)M_{22}(1,t) \\ \qquad\qquad + M_{12}(s,t), \quad \text{and} \\ N_2(s,t) = M_{21}(s,0)K(0,1)M_{22}(1,t) \\ \qquad\qquad + M_{22}(s,t). \end{cases}$$

From one of the defining identities for M, we have

$$\begin{cases} N_1(s,t) - N_1(t,t)\int_t^s d\phi(u)\cdot N_2(u,t) = 0, \text{ and} \\ N_2(s,t) - N_2(t,t)\int_t^s d\theta(u)\cdot N_1(u,t) = 0, \end{cases}$$

where $N_1(t,t) = K(t,t)$ and $N_2(t,t) = L'(t,t) + 1 = L''(t,t)$; hence it must be that

$$N_1(s,t) = K(s,t) \quad \text{and} \quad N_2(s,t) = L''(s,t) \quad \text{for } s \ge t.$$

The case where there is no restriction on $g(0)$ and $g(1)$ is treated in similar fashion. This completes our argument.

THEOREM 5.6 *The function M is determined by K, L', and L'' in accordance with the following identities:*

$$\begin{cases} M_{11}(s,t) = K(s,1)K(0,1)^{-1}L''(t,0)^* \\ \qquad\qquad - K(s,0)K(1,0)^{-1}L'(t,1)^*, \\ M_{21}(s,t) = L'(s,1)K(0,1)^{-1}L''(t,0)^* \\ \qquad\qquad - L''(s,0)K(1,0)^{-1}L'(t,1)^*, \\ M_{12}(s,t) = K(s,0)K(1,0)^{-1}K(1,t) \\ \qquad\qquad - K(s,1)K(0,1)^{-1}K(0,t), \\ M_{22}(s,t) = L''(s,0)K(1,0)^{-1}K(1,t) \\ \qquad\qquad - L'(s,1)K(0,1)^{-1}K(0,t). \end{cases} \tag{5.11}$$

PROOF. This computational result depends upon Theorem 5.4 and the properties of M which we have indicated in the paragraph pre-

ceding that Theorem. One observes first the matric identity.

$$M(s,1)\begin{pmatrix} K(1,0) & -K(1,1) \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} K(s,0) & -K(s,1) \\ L''(s,0) & -L'(s,1) \end{pmatrix},$$

from which it follows immediately that

$$M(s,1) = \begin{pmatrix} K(s,0) & -K(s,1) \\ L''(s,0) & -L'(s,1) \end{pmatrix} \cdot \begin{pmatrix} K(1,0)^{-1} & K(1,0)^{-1}K(1,1) \\ 0 & 1 \end{pmatrix}.$$

Making use of the relationships $M_{11}(1, t) = M_{22}(t, 1)^*$, *etc.*, one next constructs $M(1, t)$. The formulas (5.11) then follow from the fact that $M(s, t) = M(s, 1)M(1, t)$. We omit the details.

The following theorem we state without proof. The results follow from the preceding theorems, and from our theorems about quasi-harmonic operators [10].

THEOREM 5.7 *Let ψ be a function of bounded variation from* [0, 1] *to* $B\{S_0, Q_0\}$ *such that, for each number z, the function F_z,*

$$F_z(r) = \begin{pmatrix} 0 & \phi(r) \\ \theta(r) - z\psi(r) & 0 \end{pmatrix},$$

satisfies the condition imposed on $F = F_0$ by postulate IV, *and define $M(s, t; z) = {}_s\Pi^t\{I + dF_z\}$. Then the equation,*

$$Tf(s) = \int_0^1 K(s,t)\cdot d\psi(t)\cdot f(t), \tag{5.12}$$

defines a linear transformation T from S to S with the properties:

(i) *for each f_1, f_2 in $S \times S$, $Q(f_1, Tf_2) = \int_0^1 Q_0(f_1, d\psi\cdot f_2)$.*

(ii) *T belongs to $B\{S, Q\}$ and $|T| \leq V|K|_0$, where $V = \int_0^1 |d\psi|$ and $|K|_0$ is the least upper bound of $|K(r, r)|$ for r in* [0, 1].

(iii) *if z is a number then there is a non-trivial solution f of the equation $f = zTf$ only in case $M_{21}(0, 1; z)$ is not reversible.*

(iv) *in case $M_{21}(0, 1; z)$ is not reversible, each non-trivial f such that $f = zTf$ is of the form $f(s) = M_{11}(s, 0; z)x$ where x is a non-zero point of S_0 such that $M_{21}(1, 0; z)x = 0$.*

(v) *the function L in $B(K)$ such that $T \backsim L$ as in Corollary* 3.1c *is defined by the equation $L(s, t) = \int_0^1 K(s, u)\cdot d\psi(u)\cdot K(u, t)$.*

THEOREM 5.8 *Suppose, in Theorem* 5.7, *that z is a number such that $M_{21}(1, 0; z)^{-1}$ is in $B\{S_0, Q_0\}$. Then the following statements are true:*

(i) $M_{21}(0, 1; z)^{-1} = M_{12}(1, 0; z) - M_{11}(1, 0; z)M_{21}(1, 0; z)^{-1}M_{22}(1, 0; z)$.

(ii) *$[1 - zT]^{-1}$ is in $B\{S, Q\}$ and the function K_z in $B(K)$ such that $[1 - zT]^{-1} \backsim K_z$ as in Corollary* 3.1c *is given by*

$$K_z(s,t) = \begin{cases} -M_{11}(s, 0; z)M_{21}(1, 0; z)^{-1}M_{22}(1, t; z) & \text{if } s \leq t, \text{ and} \\ M_{11}(s, 1; z)M_{21}(0, 1; z)^{-1}M_{22}(0, t; z) & \text{if } s \geq t. \end{cases}$$

(iii) *if f, g is in $S \times S$ then, in order that*

$$f(s) - z\int_0^1 K(s,t)\cdot d\psi(t)\cdot f(t) = g(s) \quad \text{for each } s \text{ in } [0,1],$$

it is necessary and sufficient that

$$f(s) = g(s) + z\int_0^1 K_z(s,t)\cdot d\psi(t)\cdot g(t) \quad \text{for each } s \text{ in } [0,1].$$

(iv) *if h is a function of bounded variation from* [0, 1] *to $\{S_0, Q_0\}$ then the only ordered pair f, g of functions, of bounded variation from* [0, 1] *to $\{S_0, Q_0\}$, such that $g(0) = g(1) = 0$ and*

$$\begin{cases} f(b) - f(a) + \int_a^b d\phi\cdot g = 0 \quad \text{and} \\ \int_a^b d\theta\cdot f + g(b) - g(a) = z\int_a^b d\psi\cdot f + h(b) - h(a) \end{cases} \tag{5.13}$$

for each subinterval $[a, b]$ of [0, 1], *is the ordered pair f, g given by*

$$\begin{cases} f(s) = \int_0^1 K_z(s,t)\cdot dh(t) \quad \text{and} \\ g(s) = \int_0^s L_z''(s,t)\cdot dh(t) + \int_s^1 L_z'(s,t)\cdot dh(t), \end{cases} \tag{5.14}$$

where L_z' and L_z'' are the functions defined by

$$\begin{cases} L_z'(s,t) = -\int_0^s d\theta(u)\cdot K_z(u,t) + z\int_0^s d\psi(u)\cdot K_z(u,t), \\ L_z''(s,t) = \int_s^1 d\theta(u)\cdot K_z(u,t) - z\int_s^1 d\psi(u)\cdot K_z(u,t). \end{cases}$$

PROOF. Assertion (i) is an immediate consequence of the fact that $M(1, 0; z)^{-1} = M(0, 1; z)$ in $B\{S_0^2, Q_0^2\}$. Next one shows that (5.14) gives the only solution f, g of (5.13) such that $g(0) = 0$ and $f(0) = \int_0^1 K_z(0, t) \cdot dh(t)$: this is accomplished by proceeding exactly as in our proof of Theorem 5.5, and we omit the somewhat repetitious details.

Now we show that if f belongs to S then

$$Q(f[\,.\,], K_z[s, \,.\,]^* x) = Q_0(f[s] + z \int_0^1 K_z[s, t] \cdot d\psi[t] \cdot f[t], x)$$

for each s, x in $[0, 1] \times S_0$: using the equations,

$$\begin{cases} M_{22}(r, t; z)^* = 1 + \int_r^t d\phi(u) \cdot M_{21}(r, u; z)^* \quad \text{and} \\ M_{21}(r, t; z)^* = \int_r^t d[\theta(u) - z\psi(u)]^* \cdot M_{22}(r, u; z)^*, \end{cases}$$

for the cases $r = 0$ and $r = 1$, a short computation shows that $Q(f[\,.\,], K_z[s, \,.\,]^* x)$ has the value

$$Q_0(f[s], R[s]^* x) + z \int_0^1 Q_0(f[t], d\psi[t]^* \cdot K_z[s, t]^* x),$$

where $R(s)$ denotes the transformation

$$M_{11}(s, 1; z) M_{21}(0, 1; z)^{-1} M_{21}(0, s; z) + M_{11}(s, 0; z) M_{21}(1, 0; z)^{-1} M_{21}(1, s; z);$$

using the equations,

$$M(s, 1; z) = M(s, 0; z) M(0, 1; z) \quad \text{and}$$
$$M(1, s; z) = M(1, 0; z) M(0, s; z),$$

together with assertion (i) of this theorem, one finds that $R(s) = 1$ and thus we have the desired expression for the integral $Q(f[\,.\,], K_z[s, \,.\,]^* x)$.

On the other hand, it is easy to see that if f, g is a solution of (5.13) then

$$Q(f[\,.\,], K_z[s, \,.\,]^* x) - z \int_0^1 Q_0(d\psi[t] \cdot f[t], K_z[s, t]^* x)$$
$$= Q_0(K_z[s, 0] g[0] - K_z[s, 1] g[1] + \int_0^1 K_z[s, t] \cdot dh[t], x);$$

this, together with the results of the two preceding paragraphs, establishes assertion (iv) of this theorem.

We now establish (iii). Suppose each of f and g is in S and $z \neq 0$, and let $f_0 = 1/z[f - g]$. Combining the results of Theorem 5.5 and assertion (iv) of this theorem, we see that for each function h of bounded variation from $[0, 1]$ to $\{S_0, Q_0\}$ the equation

$$f_0(s) = \int_0^1 K_z(s, t) \cdot dh(t)$$

is equivalent to

$$f_0(s) - z \int_0^1 K(s, t) \cdot d\psi(t) \cdot f_0(t) = \int_0^1 K(s, t) \cdot dh(t)$$

Assertion (iii) follows by taking $h(t) = \int_0^t d\psi(u) \cdot g(u)$.

Now, (iii) shows that $[1 - z\,T]$ maps S onto S reversibly, and this transformation is closed since it is continuous. It follows that $[1 - z\,T]^{-1}$ belongs to $B\{S, Q\}$. By the second paragraph of this proof we see that $[1 - z\,T]^{-1}$ corresponds to K_z in the indicated manner, so that K_z must belong to $B(K)$. Thus (ii) is established, and this completes our proof.

Remark 1. Under the supposition that $\{S_0, Q_0\}$ is n-dimensional complex Euclidean space, we make two observations:

(i) In Theorems 5.7 and 5.8, $M_{21}(1, 0; z)$ is an $n \times n$ matrix of entire (analytic) functions so that its determinant cannot be zero at each point z of any set having a limit point (in the "finite" plane)—this information is readily translated into facts about the spectrum of the transformation T.

(ii) The subject matter of this chapter might be regarded as a natural extension to n dimensions of the classical (one-dimensional) Sturm-Liouville differential equation system,

$$\left.\begin{aligned} (pf')' - qf + r = 0 \\ p(0)f'(0) = p(1)f'(1) = 0 \end{aligned}\right\},$$

where we make the identification

$$\phi(t) = \int_0^t 1/p, \quad \theta(t) = \int_0^t q, \quad h(t) = \int_0^t r.$$

Because of the discontinuities allowed by our use of mean integrals, our results are significant even in the one-dimensional case.

Remark 2. In this chapter we have chosen, for the purpose of illustration, to consider the system (5.1) subject primarily to the boundary condition $g(0) = g(1) = 0$. Certain other conditions yield to similar analysis in terms of linear subspaces of S. For example, the boundary condition $f(0) = g(1) = 0$ is analyzed in the linear subspace of S to which f belongs only in case $f(0) = 0$: Theorem 3.5 provides the corresponding kernel K' in the form

$$K'(s, t) = M_{11}(s, 1)M_{11}(0, 1)^{-1}M_{12}(0, t) \quad \text{if } s \geq t,$$

obtained, after some simplification, from (5.9) in accordance with Remark 3 following Theorem 3.5.

6. Other linear integral equations

It seems natural to inquire further about integral equations of the form occurring in part (iii) of Theorem 5.8, and we now state, as theorems, a few facts which are easily established on the basis of the theorems of Chapters 2 and 3.

Suppose $[a, b]$ is a number interval and $\{K, [a, b], S, Q\}$ is a kernel system, and suppose that

(i) $|K(r, r)|$ is bounded for r in $[a, b]$, with least upper bound $|K|_0$.

(ii) for each t in $[a, b]$ and each x in S_0, the function $K(\cdot, t)x$ is quasi-continuous from $[a, b]$ to $\{S_0, Q_0\}$.

(iii) ψ is a function of bounded variation from $[a, b]$ to $B\{S_0, Q_0\}$, with total variation $V = \int_a^b |d\psi|$.

Theorem 6.1 *Each function in S is quasi-continuous from $[a, b]$ to $\{S_0, Q_0\}$.*

Theorem 6.2 *The equation $Tf(s) = \int_a^b K(s, u)\cdot d\psi(u)\cdot f(u)$, interpreted as meaning that for each x in S_0*

$$Q_0(Tf[s], x) = \int_a^b Q_0(d\psi[u]\cdot f[u], K[u, s]x), \tag{6.1}$$

defines an element T of $B\{S, Q\}$, and

$$|T| \leq V|K|_0.$$

Theorem 6.3 *For each f_1, f_2 in $S \times S$,*

$$Q(Tf_1, f_2) = \int_a^b Q_0(d\psi\cdot f_1, f_2). \tag{6.2}$$

Theorem 6.4 *The function L in $B(K)$, such that $T \backsim L$ as in Corollary 3.1c, is given by $L(s, t) = \int_a^b K(s, u)\cdot d\psi(u)\cdot K(u, t)$; this to be interpreted as meaning that for each x, y in $S_0 \times S_0$*

$$Q_0(L[s, t]x, y) = \int_a^b Q_0(d\psi[u]\cdot K[u, t]x, K[u, s]y). \tag{6.3}$$

Theorem 6.5 *Suppose z is a number, $[1 - zT]^{-1}$ belongs to $B\{S, Q\}$, and K_z is the function in $B(K)$ such that $[1 - zT]^{-1} \backsim K_z$. Then both the following hold:*

$$\begin{cases} K_z(s, t) - z\int_a^b K(s, u)\cdot d\psi(u)\cdot K_z(u, t) = K(s, t) \\ K_z(s, t) = K(s, t) + z\int_a^b K_z(s, u)\cdot d\psi(u)\cdot K(u, t) \end{cases} \tag{6.4}$$

when interpreted in the sense of the integral in Theorem 6.4.

Theorem 6.6 *Under the suppositions of Theorem* 6.5, *if f, g is in $S \times S$ then the following equations are equivalent:*

$$\begin{cases} f(s) - z\int_a^b K(s, u)\cdot d\psi(u)\cdot f(u) = g(s) \\ f(s) = g(s) + z\int_a^b K_z(s, u)\cdot d\psi(u)\cdot g(u) \end{cases} \tag{6.5}$$

when interpreted in the sense of the integral in Theorem 6.2.

Remark. The preceding theorems provide a setting for the analysis of certain other integral equations. For example, suppose L_1 is any element of $B(K)$, T_1 is the element of $B\{S, Q\}$ which is defined by $T_1 \backsim L_1$, and T_2 is defined by:

$$T_2f(s) = \int_a^b L_1(s, u)\cdot d\psi(u)\cdot f(u). \tag{6.6}$$

in the sense of Theorem 6.2. Then we have

$$Q_0(T_2f[s], x) = \int_a^b Q_0(d\psi[u]\cdot f[u], L_1^*[u, s]x), \tag{6.7}$$

from which it follows that for all f and g in S

$$Q(T_2f, g) = \int_a^b Q_0(d\psi\cdot f, T_1^*g) = Q(Tf, T_1^*g) = Q(T_1Tf, g). \tag{6.8}$$

Thus we see that $T_2 = T_1T$, where T is as defined in Theorem 6.2.

7. An example from function theory

Let E be a connected domain (*i.e.*, a connected open set) in the complex number plane.

DEFINITION 7.1 *A regular function* from E to $\{S_0, Q_0\}$ is a function f from E to $\{S_0, Q_0\}$ such that for each point x in S_0 the function g—$g(z) = Q_0(f[z], x)$ for z in E—is a regular (analytic) function in the domain E.

We suppose in this chapter that P is a weakly continuous function from E to $B\{S_0, Q_0\}$, *i.e.*, for each x in S_0 the function $Q_0(x, P[\cdot]x)$ is continuous from E to the numbers, and that the following postulates hold:

I. For each closed bounded set D lying in E, $m(D)$ and $M(D)$ are positive numbers such that, for all z in D and all x in S_0,

$$m(D)\ Q_0(x, x) \leq Q_0(x, P[z]x) \leq M(D)\ Q_0(x, x).$$

II. There exists a non-trivial regular function f from E to $\{S_0, Q_0\}$ such that $Q_0(f[\cdot], P[\cdot]f[\cdot])$ is integrable over E, with $\iint_E Q_0(f, Pf)$ understood to be the least upper bound of integrals of $Q_0(f, Pf)$ over closed bounded sets lying in E which are closures of subdomains of E.

LEMMA 7.1 *If f is a regular function from E to $\{S_0, Q_0\}$, then f is continuous from E to $\{S_0, Q_0\}$.*

PROOF. Suppose f is a regular function from E to $\{S_0, Q_0\}$ and b is a point of E. There exists a positive number r such that if $|z - b| \leq r$ then z belongs to E. Since f is weakly continuous, there is a positive number M such that $\|f(z) - f(b)\| \leq M$ for all z in E such that $|z - b| \leq r$. Hence, for x in S_0 and $Q_0(x, x) = 1$,

$$|Q_0(f[z] - f[b], x)| \leq M \quad \text{for} \quad |z - b| \leq r;$$

by Schwarz's lemma, if x is in S_0 and $Q_0(x, x) = 1$ then

$$|Q_0(f[z] - f[b], x)| \leq M\,|z - b|/r \quad \text{for} \quad |z - b| \leq r.$$

Thus $\|f(z) - f(b)\| \leq M\ |z - b|/r$ for $|z - b| \leq r$, so that f is continuous at b.

Remark. If f is a regular function from E to $\{S_0, Q_0\}$ then $f(E)$ lies in a separable subspace of $\{S_0, Q_0\}$: in particular, if N is a countable subset of E which is dense in E then, since f is continuous, $f(E)$ is a subset of the closure in $\{S_0, Q_0\}$ of the linear manifold generated by $f(N)$.

LEMMA 7.2 *If f is a regular function from E to $\{S_0, Q_0\}$ and b is a point of E and R is a positive number such that if $|z - b| \leq R$ then z is in E, then*

$$\pi R^2 Q_0(f[b]), f[b]) \leq \int_0^R \int_0^{2\pi} Q_0(f[b + re^{i\theta}], f[b + re^{i\theta}])r\ dr\ d\theta. \tag{7.1}$$

PROOF. Elementary computation with power series shows that for each x in S_0

$$\pi R^2\,|\,Q_0(f[b], x)\,|^2 \leq \int_0^R \int_0^{2\pi} |\,Q_0(f[b + re^{i\theta}], x)\,|^2\, r\ dr\ d\theta. \tag{7.2}$$

Now, let $\{x_n\}$ be a countable orthonormal set in $\{S_0, Q_0\}$ such that $f(E)$ is a subset of the closure in $\{S_0, Q_0\}$ of the linear manifold generated by $\{x_n\}$. Since $Q_0(f[\cdot], f[\cdot])$ is continuous and

$$Q_0(f[z], f[z]) = \sum |\,Q_0(f[z], x_n)|^2 \quad \text{for each } z \text{ in } E, \tag{7.3}$$

it follows by *Dini's Theorem* (see, *e.g.*, p. 47 of [3]) that the series in (7.3) converges uniformly for $|z - b| \leq R$. Thus (7.1) follows from (7.2), and the lemma is proved.

DEFINITION 7.2 If D is a point set in the plane and R is a positive number then $D \dot{+} R$ denotes the point set to which z belongs only in case there is a point w in D such that $|z - w| \leq R$.

THEOREM 7.1 *If f is a regular function from E to $\{S_0, Q_0\}$ such that $\iint_E Q(f, Pf)$ exists, D is a closed bounded set lying in E, and R is a positive number such that $D \dot{+} R$ is a subset of E, then*

$$\iint_E Q_0(f, Pf) \geq \pi R^2 m(D \dot{+} R) Q_0(f[z], f[z]) \quad \text{for all } z \text{ in } D. \tag{7.4}$$

Proof is omitted, since this follows directly from the preceding lemma and the postulates I and II for P.

THEOREM 7.2 *Let S be the set of all regular functions f from E to $\{S_0, Q_0\}$ such that $\iint_E Q_0(f, Pf)$ exists. Then S is a linear space, and the equation,*

$$Q(f, g) = \iint_E Q_0(f, Pg) \quad \text{for all } f, g \text{ in } S \times S \tag{7.5}$$

defines an inner product Q for S.

Proof is omitted. We remark that if each of f and g is a regular function from E to $\{S_0, Q_0\}$ then $Q_0(f[\cdot], P[\cdot]g[\cdot])$ is a continuous function from E to the numbers.

THEOREM 7.3 *The space $\{S, Q\}$ is a complete inner product space in which there exists a kernel.*

PROOF. Suppose $\{f_n\}$ is a convergent sequence in $\{S, Q\}$. By Theorem 7.1, $\{f_n(z)\}$ converges in $\{S_0, Q_0\}$ uniformly for z in each closed bounded set lying in E, and we see that the limit function f is regular from E to $\{S_0, Q_0\}$. Let c be a positive number, and N be a positive integer such that $Q(f_i - f_j, f_i - f_j) < c$ for $i, j > N$. Then, if D is a bounded set lying in E and D is the closure of a subdomain of E,

$$\iint_D Q_0(f_i - f_j, P[f_i - f_j]) < c \text{ for } i, j > N,$$

so that we have

$$\iint_D Q_0(f - f_j, P[f - f_j]) \leq c \text{ for } j > N;$$

thus we see that f is in S and $Q(f - f_j, f - f_j) \leq c$ for $j > N$. Therefore $\{S, Q\}$ is complete and, by Theorem 2.6, there is a kernel in $\{S, Q\}$.

Remark 1. If $\{S_0, Q_0\}$ is a separable space, it follows from Theorem 2.9 that $\{S, Q\}$ is separable.

Remark 2. Our supposition that P is weakly continuous can be relaxed in many ways which will suggest themselves to the reader. The principal property to be retained is that the space S defined in Theorem 7.2 should be linear. Thus we see that weak continuity can be replaced by any condition which ensures the following postulate:

III. If f and g are regular functions from E to $\{S_0, Q_0\}$ such that each of $\iint_E Q_0(f, Pf)$ and $\iint_E Q_0(g, Pg)$ exists and D is a bounded subset of E which is the closure of a subdomain of E, then the integral $\iint_D Q_0(f + g, P[f + g])$ exists.

Illustration. We now give a realization of the situation treated in this chapter. Let $\{S_0, Q_0\}$ be two-dimensional complex Euclidean space (as at the end of Chapter 1), E be the set of all complex numbers z such that $|z| < 1$, and P be defined by:

$$P(z) = \begin{pmatrix} [1 - |z|] & \tfrac{1}{2}[1 - |z|^2] \\ \tfrac{1}{2}[1 - |z|^2] & \tfrac{1}{2}[1 - |z|^2] \end{pmatrix} \tag{7.6}$$

for z in E.

If $f(z) = \{\sum a_n z^n, \sum b_n z^n\}$ for z in E, then in order that f belong to S it is necessary and sufficient that the series

$$\sum_{n=0}^{\infty} \frac{2\pi}{n+1}\left\{\frac{|a_n|^2}{2n+3} + \frac{2Re(a_n b_n^*)}{2n+4} + \frac{|b_n|^2}{2n+5}\right\} \tag{7.7}$$

converge; if (7.7) converges, its limit is $Q(f, f)$. A maximal orthonormal set $\{\phi_n\}$ in the space $\{S, Q\}$ is obtained as follows: for each non-negative integer n, let $\phi_{2n}(z) = \{a_{2n} z^n, b_{2n} z^n\}$ and $\phi_{2n+1}(z) = \{a_{2n+1} z^n, b_{2n+1} z^n\}$, where

$$\begin{cases} a_{2n} = \left(\dfrac{(n+1)(2n+3)}{2\pi}\right)^{1/2}, \\ a_{2n+1} = (2n+3)\left(\dfrac{(n+1)(2n+5)}{2\pi}\right)^{1/2}, \\ b_{2n} = 0, \\ b_{2n+1} \\ \quad = -(2n+4)\left(\dfrac{(n+1)(2n+5)}{2\pi}\right)^{1/2}. \end{cases} \tag{7.8}$$

Now, using Theorem 2.8, one finds a series expansion for $K(s, t)$ in powers of (st^*), the coefficient of $(st^*)^n$ being

$$\frac{1}{2\pi}\begin{pmatrix} [n+1][2n+3][2n+4]^2 \\ -[n+1][2n+3][2n+4][2n+5] \\ -[n+1][2n+3][2n+4][2n+5] \\ [n+1][2n+4]^2[2n+5] \end{pmatrix}. \tag{7.9}$$

Elementary computation with power series gives us $K(s, t) = M(st^*)$ where, for z in E,

$$M(z) = \frac{2}{\pi[1-z]^5}\begin{pmatrix} 6[z^2 + 5z + 2] \\ -3[z^2 + 10z + 5] \\ -3[z^2 + 10z + 5] \\ 2[z^2 + 13z + 10] \end{pmatrix}. \tag{7.10}$$

The only elements of $B\{S_0, Q_0\}$ which belong to $B\{S, Q\}$ as "constant multipliers" (see Remark preceding Theorem 3.2) are those 2×2 complex matrices A such that $A_{11} - A_{22} = A_{12} - A_{21}$.

SUPPLEMENTARY COMMENT

One interesting generalization to a more abstract setting (see Prefatory Remarks concerning the work of Pedrick, *et al.*) of the ideas developed in this paper consists in replacing our inner product space $\{S_0, Q_0\}$ in the following way. Let E be a set, X be a linear topological space with adjoint space X^*, and $\{S, Q\}$ be an inner product space of functions from E to X: a *kernel* in the space $\{S, Q\}$ is a function K from $E \times E$ such that

(i) if s, t is in $E \times E$ then $K(s, t)$ is a linear transformation from X^* into X, and

(ii) if t, y is in $E \times X^*$ then $K(\cdot, t)y$ is a member of the space S and $y(f(t)) = Q(K[\cdot, t]y, f[\cdot])$ for all f in S.

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